MATTERS ARISING

Cometary evidence against the solar companion

WILKINS¹ has concluded, from his computation of the perturbation of cometary orbits, that the Sun cannot have a fast moving neutron star as a companion. His results exclude all objects other than a massive black hole ($350 M_{\odot}$) with velocity $\gtrsim 10^2 \text{ km s}^{-1}$ as candidates for the solar companion whose existence was proposed by Harrison². In a previous paper³ I performed a closely similar calculation and concluded that an object of solar mass cannot be ruled out as a solar companion on the basis of cometary evidence, provided that its speed exceeds 20 km s^{-1} . Our results are thus in conflict by several orders of magnitude.

In equation (8) of Wilkins' paper, the perturbation of a comet's energy is given, to an order of magnitude, by the difference in the potential exerted by the companion at the end points of the comet's orbit, that is

$$\Delta E \sim \frac{Gm_c}{D} \cdot \frac{r_{\odot i} \cos \theta}{D}.$$

If m_c and D are now increased, such that the ratio m_c/D^2 remains constant, the resulting perturbation is unchanged. This means that the perturbation as calculated by Wilkins would still exist if the Sun and comet were freely falling in a uniform gravitational field; the calculation must therefore be in error.

A correct evaluation of the perturbation of the cometary orbit uses the gradient of the disturbing function due to the companion⁴, as in equation (9) of ref. 3. In this case the perturbation

$$\Delta\left(\frac{1}{a}\right) \propto Gm_c/D^2 v$$

goes to zero as D becomes large, since the velocity v scales with D .

J. G. KIRK

Max-Planck-Institut für Physik & Astrophysik,
Institut für Astrophysik,
Karl-Schwarzschild-Strasse 1, 8046
Garching bei München, FRG

WILKINS REPLIES The black hole was mentioned as only one example satisfying the luminosity criteria of Pineault¹. But its supposed distance (15,000 AU) and mass may need to be increased severalfold.

Note that Kirk uses a different reference frame to calculate ε , the specific energy (per unit mass) of the comet (K). Kirk finds ε_i relative to the instantaneous rest frame of the Sun (S), which is accelerated by the companion (C). I, however, determine ε_F relative to a fixed inertial frame, that of the Sun at apparition. These energies increase (Δ) as follows:

$$\Delta \varepsilon_1 = \int (\mathbf{f}_{CK} - \mathbf{f}_{CS}) \cdot (\mathbf{v}_K - \mathbf{v}_S) dt \quad (1)$$

$$\Delta \varepsilon_F = \int \mathbf{f}_{CK} \cdot \mathbf{v}_K dt - \int \mathbf{f}_{KS} \cdot \mathbf{v}_S dt \quad (2)$$

(In general ε_1 is the total energy of the binary system K-S referred to its centre of mass, divided by its reduced mass.) Here $\mathbf{f}_{ij}$ is the force (per unit comet mass) of body i on j . Equation (1) is the integral form of Kirk's equation (9), divested of esoteric notation. Integration by parts of equation (2) yields $\Delta \varepsilon_F = \Delta[\varepsilon_1 + \mathbf{v}_S \cdot (\mathbf{v}_K - \mathbf{v}_S) + v_S^2/2]$, which also follows from the definitions of $\varepsilon_{F,1}$. Because the cross-term vanishes for a parabolic comet and because the last term $\simeq (Gm_c/Rv_C)^2$, where $R = r_{CS}$ at apparition, is small, $\Delta \varepsilon_F \simeq \Delta \varepsilon_1$.

The discrepancy between my results and those of Kirk arose from my too-crude approximation for the first integral of equation (2), and the second integral should have been three times larger than in my equation (6). Thus I did not realize that the two integrals would cancel almost completely, leaving a difference smaller than either by about r_i/R (defined after equation (3)).

A head-on approach of C towards S, at high speed $v_C = \text{constant}$, can be calculated analytically. Introducing the semi-major axis a through $1/a = -2\varepsilon_1/Gm_S$, one finds

$$(1/a) = -0.22(3 \cos^2 \psi - 1) \times (m_C/m_S)[(r_i/R)^2 + 0(r_i^3/R^3)]R^{-1} \quad (3)$$

ψ is the polar angle from axis CS about S, of the comet's zero-energy radial infall to the Sun; the comet's distance from the Sun at a time $2R/v_C$ before apparition is denoted by $r_i = (9Gm_S/2)^{1/3}(2R/v_C)^{2/3}$. Averaging over the isotropic flux of comets, one finds that the mean value of $1/a$ is unchanged, but its r.m.s. spread due to C, equals $\sigma_C = 0.2 \times (m_C/m_S)(r_i/R)^2 R^{-1}$. Demanding that $\sigma_C < 2 \times 10^{-5} \text{ AU}^{-1}$ (as for comets

without non-gravitational forces²) and setting $g_C = Gm_C/R^2$ yields the constraint

$$(g_C/10^{-6} \text{ cm}^{-2})^{1/3} (R/10^3 \text{ AU})^{1/3} \times (30 \text{ km s}^{-1}/v_C)^2 < 0.86 \quad (4)$$

If, for example, the first two factors are each unity, this gives $v_C > 32 \text{ km s}^{-1}$, exceeding Kirk's estimate. Actually, the lower limit should be still larger because an off-side collision prolongs the epoch of closest approach and unless v_C were augmented, σ_C would increase.

The smallness of Neptune's perturbations, however, forbids a close encounter unless the companion has speeds well above 10^2 km s^{-1} . Let $T = 2R/v_C$ be the effective transit time (10^2 yr for $R = 10^3 \text{ AU}$, $v_C = 10^2 \text{ km s}^{-1}$). Neptune's known perturbations roughly limit

$$(Gm_C/R^3) \times \begin{cases} T > 170 \text{ yr} & (a) \\ (T/170 \text{ yr})^2, T < 170 \text{ yr} & (b) \end{cases} \quad (5)$$

to less³ than $1.2 \times 10^{-24} \text{ s}^{-2}$. Case (a) yields

$$R > 5.3 \times 10^4 \text{ AU} (g_C/10^{-6} \text{ cm s}^{-2}) \quad (6)$$

This excludes a close approach lasting more than 170 yr. Only a briefer encounter, case (b), permits $R \sim 10^3 \text{ AU}$, if the speed is high enough:

$$v_C > 440 \text{ km s}^{-1} (R/10^3 \text{ AU}) \times (g_C/10^{-6} \text{ cm s}^{-2})^{1/3} \quad (7)$$

This easily predominates over the cometary limit above.

My article gave a limit on R three times smaller than in equation (6) because I had adopted the allowed tidal gradient⁴ for a companion in circular orbit at $r_{CS} \leq 75 \text{ AU}$, which turns out unexpectedly to be three times larger than the value³ for $r_{CS} = 150 - 600 \text{ AU}$. Equations (5), (6) use the latter value, assuming that it has already stabilized at its large-distance limit.

Note added in proof: In work to be published I confirm that the right side of equation (4) never exceeds 0.86 for any fast encounter.

DANIEL WILKINS

Istituto di Astrofisica Spaziale del CNR,
CP 67, I-00044, Frascati, Italy

1. Wilkins, D. *Nature* **282**, 696-697 (1979).
2. Harrison, E. R. *Nature* **270**, 324-326 (1977).
3. Kirk, J. G. *Nature* **274**, 667-669 (1978).
4. Smart, W. M. *Celestial Mechanics*, 9 (Longmans, London, 1953).

1. Pineault, S. *Nature* **275**, 727-730 (1978).
2. Marsden, B. G., Sekanina, Z. & Everhart, E. *Astr. J.* **83**, 64-71 (1978).
3. Rawlins, D. & Hammerton, M. *Mon. Not. R. astr. Soc.* **162**, 261-270 (1973).
4. Rawlins, D. & Hammerton, M. *Nature* **240**, 457 (1972).