MATTERS ARISING

Cometary evidence against the solar companion

WILKINS[1] has concluded, from his computation of the perturbation of cometary orbits, that the Sun cannot have a fast moving neutron star as a companion. His results exclude all objects other than a massive black hole ($350\,M_{\odot}$) with velocity $\gtrsim 10^2\,\mathrm{km\,s^{-1}}$ as candidates for the solar companion whose existence was proposed by Harrison[2]. In a previous paper[3] I performed a closely similar calculation and concluded that an object of solar mass cannot be ruled out as a solar companion on the basis of cometary evidence, provided that its speed exceeds $20\,\mathrm{km\,s^{-1}}$. Our results are thus in conflict by several orders of magnitude.

In equation (8) of Wilkins' paper, the perturbation of a comet's energy is given, to an order of magnitude, by the difference in the potential exerted by the companion at the end points of the comet's orbit, that is

$$\Delta E \sim \frac{Gm_c}{D}\cdot\frac{r_{\odot i}\cos\theta}{D}.$$

If m_c and D are now increased, such that the ratio m_c/D^2 remains constant, the resulting perturbation is unchanged. This means that the perturbation as calculated by Wilkins would still exist if the Sun and comet were freely falling in a uniform gravitational field; the calculation must therefore be in error.

A correct evluation of the perturbation of the cometary orbit uses the gradient of the disturbing function due to the companion[4], as in equation (9) of ref. 3. In this case the perturbation

$$\Delta\left(\frac{1}{a}\right) \propto Gm_c/D^2 v$$

goes to zero as D becomes large, since the velocity v scales with D.

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WILKINS REPLIES The black hole was mentioned as only one example satisfying the luminosity criteria of Pineault[1]. But its supposed distance (15,000 AU) and mass may need to be increased severalfold.

Note that Kirk uses a different reference frame to calculate ε, the specific energy (per unit mass) of the comet (K). Kirk finds ε_I relative to the instantaneous rest frame of the Sun (S), which is accelerated by the companion (C). I, however, determine ε_F relative to a fixed inertial frame, that of the Sun at apparition. These energies increase (Δ) as follows:

$$\Delta\varepsilon_I = \int (\mathbf{f}_{CK} - \mathbf{f}_{CS})\cdot(\mathbf{v}_K - \mathbf{v}_S)\,dt \qquad (1)$$

$$\Delta\varepsilon_F = \int \mathbf{f}_{CK}\cdot\mathbf{v}_K\,dt - \int \mathbf{f}_{KS}\cdot\mathbf{v}_S\,dt \qquad (2)$$

(In general ε_I is the total energy of the binary system K–S referred to its centre of mass, divided by its reduced mass.) Here $\mathbf{f}_{ij}$ is the force (per unit comet mass) of body i on j. Equation (1) is the integral form of Kirk's equation (9), divested of esoteric notation. Integration by parts of equation (2) yields $\Delta\varepsilon_F = \Delta[\varepsilon_I + \mathbf{v}_S\cdot(\mathbf{v}_K - \mathbf{v}_S) + \mathbf{v}_S^2/2]$, which also follows from the definitions of $\varepsilon_{F,I}$. Because the cross-term vanishes for a parabolic comet and because the last term $\simeq (Gm_c/Rv_C)^2$, where $R = r_{CS}$ at apparition, is small, $\Delta\varepsilon_F \simeq \Delta\varepsilon_I$.

The discrepancy between my results and those of Kirk arose from my too-crude approximation for the first integral of equation (2), and the second integral should have been three times larger than in my equation (6). Thus I did not realize that the two integrals would cancel almost completely, leaving a difference smaller than either by about r_i/R (defined after equation (3)).

A head-on approach of C towards S, at high speed v_C = constant, can be calculated analytically. Introducing the semi-major axis a through $1/a = -2\varepsilon_I/Gm_S$, one finds

$$(1/a) = -0.22(3\cos^2\psi - 1) \times (m_C/m_S)[(r_i/R)^2 + 0(r_i^3/R^3)]R^{-1} \qquad (3)$$

ψ is the polar angle from axis CS about S, of the comet's zero-energy radial infall to the Sun; the comet's distance from the Sun at a time $2R/v_C$ before apparition is denoted by $r_i = (9Gm_S/2)^{\frac{1}{3}}(2R/v_C)^{\frac{2}{3}}$. Averaging over the isotropic flux of comets, one finds that the mean value of $1/a$ is unchanged, but its r.m.s. spread due to C, equals $\sigma_C = 0.2 \times (m_C/m_S)(r_i/R)^2 R^{-1}$. Demanding that $\sigma_C < 2\times10^{-5}\,\mathrm{AU}^{-1}$ (as for comets without non-gravitational forces[2]) and setting $g_C = Gm_C/R^2$ yields the constraint

$$(g_C/10^{-6}\,\mathrm{cm^{-2}})^{\frac{1}{2}}(R/10^3\,\mathrm{AU})^{\frac{1}{2}} \times (30\,\mathrm{km\,s^{-1}}/v_C)^2 < 0.86 \qquad (4)$$

If, for example, the first two factors are each unity, this gives $v_C > 32\,\mathrm{km\,s^{-1}}$, exceeding Kirk's estimate. Actually, the lower limit should be still larger because an off-side collision prolongs the epoch of closest approach and unless v_C were augmented, σ_C would increase.

The smallness of Neptune's perturbations, however, forbids a close encounter unless the companion has speeds well above $10^2\,\mathrm{km\,s^{-1}}$. Let $T = 2R/v_C$ be the effective transit time (10^2 yr for $R = 10^3$ AU, $v_C = 10^2\,\mathrm{km\,s^{-1}}$). Neptune's known perturbations roughly limit

$$(Gm_C/R^3)\times\begin{cases} & ,\ T > 170\,\mathrm{yr}\quad (a)\\ (T/170\,\mathrm{yr})^2, & T < 170\,\mathrm{yr}\quad (b)\end{cases} \qquad (5)$$

to less[3] than $1.2\times10^{-24}\,\mathrm{s^{-2}}$. Case (a) yields

$$R > 5.3\times10^4\,\mathrm{AU}\,(g_C/10^{-6}\,\mathrm{cm\,s^{-2}}) \qquad (6)$$

This excludes a close approach lasting more than 170 yr. Only a briefer encounter, case (b), permits $R \sim 10^3$ AU, if the speed is high enough:

$$v_C > 440\,\mathrm{km\,s^{-1}}(R/10^3\,\mathrm{AU}) \times (g_C/10^{-6}\,\mathrm{cm\,s^{-2}})^{\frac{1}{2}} \qquad (7)$$

This easily predominates over the cometary limit above.

My article gave a limit on R three times smaller than in equation (6) because I had adopted the allowed tidal gradient[4] for a companion in circular orbit at $r_{CS} \leqq 75$ AU, which turns out unexpectedly to be three times larger than the value[3] for $r_{CS} = 150-600$ AU. Equations (5), (6) use the latter value, assuming that it has already stabilized at its large-distance limit.

Note added in proof: In work to be published I confirm that the right side of equation (4) never exceeds 0.86 for any fast encounter.

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