Theoretical difficulties with the binary pulsar?

from Malcolm MacCallum

THE agreement of the rate of change of period of the binary pulsar with the usual quadrupole formula for gravitational radiation, reported by J. H. Taylor, L. A. Fowler, and P. M. McCulloch (*Nature* **277**, 437; 1979), has been hailed as a triumph for general relativity. Unfortunately it has been known to specialists for some time that the theoretical predictions are themselves open to question.

J. Ehlers, A. Rosenblum, J. N. Goldberg and P. Havas drew attention to the unsatisfactory nature of all existing derivations of the formulae in an article in the *Astrophysical Journal* in 1976 (**208**, L77). Concerning these arguments, Taylor *et al.* remark that "our data suggest that any such inaccuracy is not very large", but there is of course the more intriguing possibility that the observations disagree with the correct general-relativistic formula. Indeed, what some specialists fear most is that the usual formula will prove correct to better than 10%, with the consequence that any future objections to non-rigorous arguments in relativistic astrophysics and cosmology will be dismissed as merely pedantic quibbles.

In deriving the standard formula, L. D. Landau and E. M. Lifschitz (in *The Classical Theory of Fields* third edition, Pergamon, 1971) wrote "In principle, all the calculations are completely analogous to those which we carried out for electromagnetic waves." It is this analogy which has given the quadrupole formula its strong intuitive appeal to physicists, but it also emphasises the underlying reasons for some of the experts' doubts, namely that it uses an essentially linear approximation in a flat space-time background when the full theory is non-linear and uses curved spaces.

Since 1976 renewed efforts have been made to reach an agreed alternative to (or confirmation of ?) the usual formula, based on more sophisticated approximations, and the Gregynog meeting* provided an opportunity to review progress. Some of the most illuminating results reported arose from model calculations using scaler waves in one dimension (whose relevance is accentuated by a recent preprint from J. M. Stewart (University of Cambridge) showing that perturbations of algebraically special space-times — which include most of the physically interesting cases — can be represented in terms of scaler potentials).

At great distances from the sources, the field is usually assumed to propagate along flat space light rays and to be expandable in inverse powers of the radial distance *r*. Stewart, speaking about definitions of mass at infinity, reminded the audience that, however small the mass involved, light rays in the field of a spherical body (a Schwarzschild metric) would diverge logarithmically from those of the corresponding flat space. M. Walker (Max-Planck-Institute, Munich) reported an example in which logarithmic terms appear in the field in the far past.

One way of relating the field to the sources is through a retarded Green's function integral. In relativity, however, such retarded solutions are not simply related to natural choices of boundary condition, such as no incoming radiation in the far past' or 'only outgoing radiation in the far future'. Walker gave an example in which the retarded solution involved incoming radiation in the far past. B. Schmidt (Max-Planck-Institute, Munich) presented a string and spring example in which the amount of incoming radiation affected the damping rate, and speculated, amusingly, that the binary pulsar measurements might mean only that if general relativity is true, the pulsar is receiving no incoming radiation. Schutz proposed avoiding the boundary conditions by assuming that at an initial instant the field external to the system, being unknown, should be taken to be zero — the average of all possible values.

The dynamics of the source has to take account of the reaction from the field. Schutz showed that in his approach the usual radiation reaction expression acquired an extra term, which he related to the need to compensate for coordinate choice effects. More detailed examinations of the internal dynamics of the sources usually follow one of two approximation schemes.

The first of these is the 'fast motion' approximation. D. Christodoulou (Max-Planck-Institute, Munich) reported on some rigorous estimates of errors in this scheme. A. Rosenblum (Temple University, Philadelphia) reported that the well-known discrepant result of Havas and Goldberg (that two bodies in a binary system spiral away from, not towards, each other) was due to neglect of non-linear terms comparable with those retained, and presented his own scattering calculations, which gave about double the result predicted by the quadrupole formula. He hopes to extend these calculations to bound systems.

The second method is the 'slow motion' technique, which involves expansion in powers of a small parameter *e*, essentially the ratio of a characteristic velocity in the system to the velocity of light. The calculations introduce terms in *er*, and are therefore valid only in a limited region. The boundary conditions for the slow motion approximation should be supplied by an outer, wave zone, solution, which itself must use the light rays of the curved space, and not a flat space approximation. Both Dixon and J. L. Anderson (Stevens Institute, Hoboken, New Jersey) have carried out model calculations of this type, joining the inner and outer regions by the method of matched asymptotic expansions. This method was introduced to relativity by W.J. Burke (*J. Math. Phys.* **12**, 401; 1971), but has been familiar in other contexts for some time, and is even known to be rigorous in certain limited circumstances. Because their calculations differed in detail, Dixon and Anderson, although in broad agreement, emphasised different aspects of the results.

Dixon's model calculation showed, contrary to some expectations, that slow motion expansions can include effects arising from tail terms of a retarded Green's function (crudely, effects propagating at a speed less than that of light), and that radiative effects can appear in even as well as odd orders in the expansion. Both authors found that the matching introduced terms in the inner region logarithmic in *e*. Anderson noted that approximations good for one feature may be bad for another (in his example, the damping and phase respectively), though I understand that at the subsequent Dublin meeting ('Current Problems in General Relativity', 2-6 July, 1979) P. Parker (Syracuse University) suggested that this could be overcome using results by Hörmander (*Acta Math.* **127**, 79; 1971).

The problem of what to approximate recurred in a final discussion excellently led by Stewart. It was agreed that intermediate concepts like energy (awkward to define in the full theory) and radiation reaction force should be avoided if possible, and attention focused on directly measurable quantities (like the binary pulsar period changes). Care would be needed to avoid tailoring approximations to produce the 'right' results, so that as many observable effects as possible should be included.

Stewart concluded the meeting by announcing a competition for a model problem, with an unambiguous numerical solution, which could act as a testbed for the different approximation schemes. It should have as many features of the full problem as possible, for example, non-linear waves, retarded coupling to a source oscillator, and back-reaction on the source. Perhaps when such a problem has been found, and solved, we will know how to tackle the binary pulsar question. Then we can decide whether the observed behaviour presents difficulties for the theory, or whether the present doubts will prove only 'theoretical' in the worse sense.

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*The third Gregynog Relativity Workshop was held at Gregynog, Newtown, Wales, on 25-28 June, 1979, under the title 'Gravitational Radiation Theory'. The organisers were B. Schutz (University College, Cardiff), G. Dixon (Churchill College, Cambridge) and M. Walker (Max-Planck-Institute, Munich).