Special relativity

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Special Relativity: The Foundation of Macroscopic Physics. By W. G. Dixon. Pp. 261. (Cambridge University Press: Cambridge and London, 1978.) £14.50.

DON'T let the title deceive you: this is not another introduction to special relativity. This is a book for those of us, working physicists or advanced students, who know a few things about relativity, thermodynamics, and electrodynamics but have not bothered to examine too closely how all these things fit together and on what sorts of assumptions they are based. Graham Dixon's aim is to isolate these assumptions and to derive from them the macroscopic laws we know so well, making full use of the spacetime viewpoint. On the whole he succeeds very well, and the book should prove rewarding to the careful and patient reader.

The first chapter studies the foundations of relativity (special and Galilean). Dixon discards the usual postulates of relativity (that the laws of physics are the same in all frames) and of the invariance of the speed of light (or for Galilean relativity, the usual velocity-addition law) in favour of two other postulates. One is the Principle of Uniformity, that in a given frame, space and time are homogeneous and space is isotropic. The other is the Extended Principle of Inertia, which not only extends the usual principle of uniform motion in the absence of forces to macroscopic bodies but also postulates that a clock's rate may depend on its speed but not on its position or direction of motion. From these two principles and a careful discussion of the nature of measurements, Dixon arrives essentially at the conclusion that the transformation law between the preferred reference frames picked out by the principles must be linear and must preserve the line element $(\delta x)^2 + (\delta y)^2 + (\delta z)^2 - C\,(\delta t)^2$, for some constant C. The case $C > 0$ is special relativity, and $C < 0$ is excluded by a rather weak argument which would have been unconvincing before the days of particle accelerators. The limiting case, $|C|\rightarrow\infty$, is actually treated separately and is, of course, Newtonian theory. (The case $C=0$ is not discussed but is in fact easy to exclude.) Later Dixon remarks that it seems as if we have got something—a fundamental constant with the dimensions of velocity—from postulates which seem to have no scale, and that this velocity may originate in the fundamental velocity $\hbar/e^2$ by way of quantum-mechanical nature of the measuring rods one uses. Although there may be something in this, a more direct explanation seems to me to be that his postulates already imply the consistency of using universal units for time and distance, and the only question is what value C has in these units. As the postulates do not restrict C, they do not themselves give a universal velocity. This restriction comes from outside the postulates, by adding that the cases $C<0$, $C=0$, and $|C|=\infty$ are contrary to experiment. But this is a minor quarrel, which should not obscure the fact that this chapter makes the useful point that special relativity and Newtonian physics need not be derived from such disparate postulates as one usually finds in elementary treatments.

This unity of treatment of relativistic and Newtonian theories is developed further in chapter 2 and then carried through the whole book. Chapter 2 begins by developing affine tensor calculus. The point of view is algebraic rather than geometric, with tensors defined by their transformation properties rather than as linear functions of vectors. Integration on arbitrary surfaces in an affine space E_n is discussed, with a full and clear treatment of antisymmetric tensors, volume elements, orientation and Stokes' theorem. Lie derivatives don't appear here but are introduced very briefly in chapter 4. In order to treat relativity and Newtonian theory in a unified manner, their respective spacetimes are discussed and fundamental tensors defined: the metric for relativity and both a 3-metric and the universal time covector in Newtonian theory. (Throughout the book Dixon uses 'Newtonian' to mean a gravity-free spacetime, which others often call Galilean.) A particularly nice feature is a demonstration of how to contract the Lorentz group to get the Galilean group.

There follows a discussion of particle and continuum dynamics, conservation laws, and an especially clear discussion of the effects of forces on mass and energy in both theories. One can also find here in chapter 3 a careful treatment of the 'Newtonian limit', which deserves a careful treatment where mass and energy are concerned.

In chapter 4, Dixon picks up his assertion that the laws of physics are seen most simply from a relativistic point of view by showing how much of thermodynamics and phenomenological hydrodynamics can be derived from exceedingly simple assumptions. His starting point is the entropy-flux four-vector s^α, which by the second law of thermodynamics satisfies $s^\alpha{}_{,\alpha} \geqslant 0$, the equality obtaining in equilibrium (but not only there). He defines a simple fluid as one in which s^α is a function only of the stress-energy tensor and the particle-flux vector, at least in and near equilibrium. From this starting point and using Lorentz invariance he shows that, in equilibrium, s^α is parallel to the flux vector and the stress-energy tensor is diag(μ,p,p,p) in the reference frame defined by s^α. Near equilibrium the usual phenomenological laws governing heat conduction, bulk viscosity, and shear viscosity come out as first-order corrections, whereas second-order corrections give the time-derivatives in the diffusion laws which are necessary to limit the diffusion to speeds less than that of light. The treatment of the second-order terms seems to follow that of W. Israel (*Ann. Phys. New York*, **100**, 310–31, 1976) to which Dixon refers, but Israel's "cross-coupling coefficients" don't appear here and it is unfortunate that Dixon does not comment on this. It is also sad that Dixon does not refer earlier in this chapter to the often-overlooked work of van Dantzig in the 1930s, who seems first to have introduced "thermasy" ($T\tau$, where τ is proper time), a variable often used by Dixon as a parameter along the world lines of his fluid.

The final chapter gives, for fluids which interact with electromagnetic fields, a treatment largely parallel to the one above, except that here of course there is no point taking any Newtonian limits. It contains a good discussion of the Abraham and Minkowski stress-energy tensors in view of the arbitrariness of any separation between 'fluid' and 'field' for a polarisable fluid. The second-order terms now include other phenomenological terms associated with the Debye and Langevin effects.

Throughout the book Dixon maintains a clear and methodical style. The book is well-produced and admirably free of misprints, the worst of which I found was a statement (p87) that no timelike vector can be expressed as a linear combination of spacelike vectors, in which it is clear from the context that "orthogonal" is intended before "spacelike". It assumes a fair degree of sophistication of the reader, and I would not send a student to it who did not already know special relativity and relativistic thermodynamics from other sources. But I would recommend it strongly for libraries and for anyone who wants to bring a little more order to that jumbled mixture of theory and phenomenology that most of us carry around in our brains. □

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