

this conventional view, but we must remember that it has not yet passed the final and most important test for a myosin, which is to show that its actin activated ATPase will cause movement. □

Coupled oscillators in chaotic modes

from N. MacDonald

THE realisation that systems with apparently simple, but non-linear, dynamics can exhibit quasi-random behaviour has provoked much activity in the past few years. The Lorenz hydrodynamical model, the launching pad for much of this work (see *News and Views* 271, 305; 1978) continues to yield rich results. Numerical investigations reveal (Shimizu & Morioka *Phys. Lett.* A66, 182, 447; 1978) that with increasing Rayleigh number one passes between successive regimes of periodic and chaotic behaviour. The topological nature of the attractors in this model has been examined in some detail (Rand *Math. Proc. Camb. Phil. Soc.* 83, 451; 1978). It is becoming clear that a wide variety of non-linear systems display analogous chaotic behaviour. It is therefore natural to seek such behaviour in models of forced or coupled non-linear oscillators, in view of their extensive application in physics, engineering and other disciplines.

It is rather characteristic of the whole subject of non-linear dynamics that the investigation of specific examples is the only way through the abundance of phenomena that emerge once the simple linear law of superposition of solutions is abandoned. There are traditionally two popular models of non-linear oscillators. The Duffing equation

$$\frac{d^2x}{dt^2} + ax + bx^3 = 0$$

originated in corrections to the ideal behaviour of a mass on a spring. The Van der Pol equation

$$\frac{d^2x}{dt^2} - c(1-x^2)\frac{dx}{dt} + x = 0$$

originated as a model of oscillations in a triode circuit. Recently the Brusselator,

$$\frac{dx}{dt} = A - Bx + x^2y,$$

$$\frac{dy}{dt} = Cx - x^2y,$$

N. MacDonald is in the Department of Natural Philosophy, University of Glasgow.

has been introduced as a crude model of chemical oscillations. (It is necessarily crude because the cubic term, taken literally, implies a collision between three molecules.)

When any of these oscillators is subjected to a sinusoidal input it shows the phenomena of simple and subharmonic entrainment. When the frequency of the input is fairly close to the natural frequency of the oscillator, or to some low multiple of that frequency, the nonlinear oscillator goes into stable periodic motion. The range implied by 'fairly close' depends on the amplitude of the input. Recent numerical studies of the Brusselator (Tomita & Kai *Phys. Lett.* A66, 91; 1978) show that at input frequency near to twice the natural frequency, for sufficiently large input amplitude, the subharmonic oscillations become unstable and are replaced by recognisably chaotic behaviour. These authors use a 'stroboscopic' method, in which one examines the trajectory of the point (x,y) at successive times separated by the period of the driving force. The continuous dynamics of the model can assimilated to the well-studied dynamics (see the review by May *Nature* 261, 459; 1976) of the onset of chaos for a discrete dynamical system. To borrow a phrase from Oscar Wilde, this is "chaos illuminated by flashes of lightning".

An investigation (Fujisaka & Yamada *Phys. Lett.* A66, 450; 1978) of sets of two or three coupled oscillators similar to the Duffing oscillator has also revealed chaotic behaviour. A practical device for demonstrating such effects in coupled nonlinear oscillators has been described (Gollub, Brunner & Danby *Science* 200, 48; 1978). They use two tunnel diodes, each connected in series to an inductor and a resistor. These units are connected in parallel, through a resistor, to a D.C. source. Oscillations are observed in the voltage across each diode, with frequencies in the ratio of two integers, the particular ratio depending on the source voltage. Next an additional resistor is used to link the two halves of the circuit. It is now possible to find aperiodic variation of the voltages across the diodes, demonstrated as a noisy power spectrum.

These results are more than a mere curiosity, or illustrations of a topic currently fashionable in mathematics. Forced and coupled oscillators have considerable relevance, for example, to a great variety of phenomena in physiology, covering time scales from seconds to weeks. Some examples may be cited here. Electromagnetic signals from the gut (Linkens, Taylor & Duthie *I.E.E.E. Trans. Biomed. Eng.* 23, 101; 1976) display complex spectra that sug-

gest that both the nature of individual oscillators and their mode of coupling require investigation. Contemporary discussion of the origins of circadian rhythms leads one to the investigation of populations of coupled oscillators. (See review by Winfree *Nature* 253, 308; 1975). The frequency spectra of the fluctuations of concentrations of blood cells of various types in the peripheral blood of collie dogs or of human patients with cyclical neutropenia suggest that one may be dealing with coupled oscillating cell lineages (see Date *et al. J. clin. Invest.* 51, 2197; 1972; Guerry *et al. J. clin. Invest.* 53, 3220; 1973 for the data, and Kurland *et al. Science* 199, 552; 1978 for some possible modes of coupling.)

The classification of possible new modes of behaviour of coupled oscillators, and the elucidation of which, if any, of these modes are universal and which depend on the specific nature of the oscillators or of their coupling, is a necessity for the full development of models for such situations. Even when it is clear that the normal functioning of the system is periodic one may wish to model pathological arrhythmic conditions (Mackey & Glass *Science* 197, 287; 1977). The numerical and experimental investigations reported above represent steps towards this goal. □



A hundred years ago

WE learn from *Harper's Weekly* that, for the purpose of prosecuting biological researches, Prof. A. Agassiz has lately completed a superb establishment near his residence at Newport, wherein every device that experience could suggest has been brought to bear for the convenience of investigators. A building 45 by 25 feet has been erected on the side of a bay making up from the entrance to Newport Harbour, and provided with the purest of sea-water by means of a steam-pump, which keeps a tank constantly filled. The tables are covered with a series of tiles of different colours, so that the minute animals of different shades can be the more readily overhauled when emptied upon them. The shelves in the laboratory are all of glass, the tanks are of slate, the conducting pipes are of iron, lined with a composition of rubber, which it is believed will protect them against corrosion. These tables are all well lighted, and are available for students, whom Mr. Agassiz invites to share his facilities.

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