

of primary and secondary waves in developing embryos does not have the logical status of a proved theorem since he has not as yet published the full mathematical proof. Rather, his use of catastrophe theory in a description of differentiation gives rise to the hypothesis that such waves occur and, with the additional postulate of a temporal periodicity of state in the tissue, this application suggests how spatially periodic structures such as somites may arise.

These hypotheses have stimulated experimental investigation, which is a major purpose of model-building. Furthermore, Zeeman's treatment of differentiation has the additional virtue of providing a unitary field description of a process which is often erroneously and misleadingly described in terms of separate spatial and temporal mechanisms. In a subject such as developmental biology, which has barely begun to come to grips with its central problem of morphogenesis in terms of models, it is more important to get the correct qualitative treatment than to attempt quantitative precision.

It is far too early to decide whether or not catastrophe theory will be of major value in biology. That it provides useful and accurate descriptions of certain physical processes is now beyond question. More generally, the context for catastrophe theory is topology, and topological thinking has been of immense value in the understanding of many physical phenomena. It seems highly probable that the topological approach will prove invaluable in the study of biological processes as well, but this is an approach that can only be learned slowly, with trial and error. Zahler and Sussman have presented some valid criticisms of applied catastrophe theory, but their over-reaction is unfortunate. It leads them into exaggeration and wholesale rejection of very useful propositions.

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SIR,—The case in favour of catastrophe theory rests not on speculative models in the social sciences, but on successful applications to the physical sciences. In 1975 and 1976 there appeared approximately 42 papers applying catastrophe theory to physics, nine to biology, and 14 others: Sussmann and Zahler's criticisms deal almost entirely with one sociological paper, two on biology, and one model taken from two popular articles and a paragraph in a conference report. They do not hesitate to extend their conclusions to areas they have not studied: "we anticipate that

the results of an extended search (covering biology, linguistics, physics, or psychology) will be similar (that is negative)" from (Sussmann, H. J. & Zahler, R. S. *Proceedings of the 1976 biennial meeting of the Philosophy of Science Association, Chicago*, in press). Tim Poston and I have written a book (Poston, T. & Stewart, I. N. *Catastrophe theory and its applications*, Pitman, London, 477 pp.), due in print early in 1978, documenting quantitative applications in the sciences, which casts severe doubt on their conclusions. A major plank in their case—allegation of a repeated mathematical error—is refuted by Poston (*Mathematics Report*, Battelle Geneva (in press)). Their reliability may be judged by their statement: "Stewart repeats the untrue assertion that Zeeman's embryological predictions have been 'recently verified by experiment'". What I wrote was: ". . . with the prediction that slowing down the chemical reactions of the primary wave would lead to the formation of fewer somites, an effect recently verified by experiment". Which happens to be true.

Similar misinterpretations vitiate many of Sussmann and Zahler's criticisms, rendering them analogous to disproving Pythagoras' theorem by exhibiting a triangle that is not right-angled. With the exception of their discussion of the nerve impulse model, few of their criticisms are conclusive, and some are simply wrong. Others are problems of general mathematical modelling, which can usually be resolved by reference to current scientific practice. Sussmann and Zahler's charges go considerably beyond anything they have correctly substantiated.

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SIR,—It would be a pity if the strong attack by Zahler and Sussman on some biological and sociological models based on catastrophe theory, (27 October, page 759) were to mislead readers into thinking that such new and beautiful mathematics has no useful application in any science. The fact is that in this laboratory catastrophe theory is being employed in the development of new concepts, in the explanation and prediction of phenomena, and in the design of experiments, in two areas of physics.

The first is short wave optics (and quantum mechanics) where Thom's theory classifies the forms of focal surfaces (caustics) and makes it possible to give a precise description of the finest detail in the associated diffraction patterns (Arnol'd, V. I. 'Critical points of smooth functions and their normal forms' *Uspekhi Mat Nauk*

(translation: *Russian Mathematical Surveys*) 30, 1–75 (1975); Berry, M. V. 'Waves and Thom's Theorem' *Adv. in Phys.* 25, 1–26 (1976); Duistermaat, J. J. 'Oscillatory integrals, Lagrange immersions and unfolding singularities' *Comm Pure App Math* 27, 207–281 (1974)). The classification describes caustics that are 'structurally stable', that is those whose forms survive perturbation. This makes catastrophe theory particularly suited to the optics of nature rather than artefacts such as microscopes and telescopes whose focussing is dominated by cylindrical symmetry.

We have made progress in understanding the optics of irregular water droplet 'lenses' (Berry, M. V. 'Waves and Thom's Theorem' *Adv. in Phys.* 25, 1–26 (1976); Nye, J. F. 'Optical caustics in the near field from liquid drops' (submitted to *Proc. Roy. Soc.*), the fine structure of swimming pool caustics (Berry, M. V. & Nye, J. F. 'Fine structure in caustic junctions' *Nature* 267, 34–6 (1976)), atom scattering by crystal surfaces (Berry, M. V. 'Cusped rainbows and incoherence effects in the rippling-mirror model for particles scattering from surfaces'. *J. Phys. A* 8, 566–84 (1975)) and the statistics of twinkling starlight (Berry, M. V. 'Focusing and twinkling: critical exponents from catastrophes in non-Gaussian random short waves' (*J. Phys. A*, in press)). This last application (which has proved peculiarly resistant to more conventional forms of analysis) makes essential use of the enormous extension of Thom's classification being developed by Arnol'd (Arnol'd, V. I. 'Critical points of smooth functions and their normal forms' *Uspekhi Mat Nauk* (translation: *Russian Mathematical Surveys*) 30, 1–75 (1975)) in the Soviet Union.

The other area is fluid mechanics, where the elliptic umbilic suggested the design of the 'sixroll mill' (Berry, M. V. & Mackley, M. R. 'The sixroll mill: unfolding an unstable persistently extensional flow'. *Phil. Trans. Roy. Soc. (London)* 287, 1–16 (1977)), a device for studying the effects of dissolved long-chain molecules on the flow of Newtonian fluid. The mill produces a sequence of flows with fully describable instabilities, and addition of polymer is dramatically revealed by changes in the topology of the pattern of streamlines. This specialised application has now been generalised (Thorndike, A. S., Cooley, C. R. and Nye, J. F. 'The structure and evolution of vector fields and other flow fields' (submitted to *J. Phys. A*)) into a comprehensive theory of flow patterns, which has already given insight into the structure of the geostrophic wind and the move-