matters arising

A precise definition of chaos

MAY[1] has recently reviewed work on the dynamics of simple recurrence relations. If $x_{n+1} = f(a; x_n)$ where a is a parameter, the system exhibits different behaviours for different values of a. In many cases, there is a range of behaviour characterised by three properties:

(1) Not all solutions are periodic;
(2) Asymptotic stability need not obtain;
(3) Cycles of all periods may occur.

Li and Yorke[2] show that (1), (2) and (3) necessarily follow if a 3-cycle exists and these authors term such behaviour 'chaotic'. They also show that a 5-cycle may exist without there being a 3-cycle.

The Ukrainian author Sharkovsky[3] proves a more general result, which deserves to be more widely known. Let $n_1 \prec n_2$ if $n_1 \neq n_2$ and the existence of an n_1-cycle implies the existence of an n_2-cycle. Sharkovsky shows that

$$3 \prec 5 \prec 7 \prec 9 \prec \ldots$$
$$\prec 2.3 \prec 2.5 \prec 2.7 \ldots$$
$$\prec 2^2.3 \prec 2^2.5 \prec 2^2.7 \prec \ldots$$
$$\prec 2^3.3 \prec 2^3.5 \prec 2^3.7$$
$$\prec \ldots \prec \ldots 2^3 \prec 2^2 \prec 2 \prec 1 \qquad (1)$$

He also demonstrates that there exist recurrence relationships with $(2m+1)$-cycles but not with $3, 5, \ldots, (2m-1)$-cycles, for any natural number $m = 2, 3, 4, \ldots$.

In a subsequent paper[4], Sharkovsky classifies the behaviour of the system in terms of the topological properties of the set C of all points belonging to cycles. If C is a closed set, then all initial values x_0 lead to asymptotically periodic behaviour. This occurs when there are only finitely many cycles (necessarily 2^n-cycles by relation (1)), and in some instances where there are infinitely many cycles all with orders of the form 2^n. If C is not a closed set the number of x_0 not leading to asymptotic periodicity is uncountably large. This occurs whenever there are cycles other than 2^n-cycles and, in some cases, where all the 2^n-cycles occur but only those.

In the first of these instances, requirements (1) and (2) are satisfied, but not necessarily (3). It is not known whether this is true in the second instance, which, however, we conjecture to be structurally unstable to changes in the value of a.

We suggest that requirements (1) and (2) be regarded as defining chaos, but that (3) not be regarded as necessary. This would entail some divergence from the nomenclature of certain authors (for example, May[1]), but it is a consistent convention and one in accord with everyday usage.

A detailed, though still incomplete, list of Sharkovsky's papers is provided by Sibirsky[5]. We also draw attention to the work of Barna[6], Berg[7] and Coppel[8].

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Heat-transfer enhancement with electric fields

CHANGES in heat transfer brought about by electric fields, as reported by Asakawa[1], have been observed for many years. The discovery of the phenomenon is generally attributed to Senftleben and Braun[2] who observed the systematic enhancement of heat transfer in various gases contained between a cylindrical surface and a coaxial wire when electric fields greater than about 42 kV cm^{-1} were produced at the surface of the wire. The effect was mentioned as early as 1709, however[3].

The results reported by Asakawa seem primarily to be the retardation of heat transfer, but in fact seem similar to those expected on the basis of simple inductive heating. It is difficult to quantify heat transfer results with transient experiments, and is generally more informative to rely on steady-state (for example, calorimetric) measurements. Although it is apparently tempting to suggest that the phenomena observed by Asakawa involve material changes in the material (fluid), there is no evidence for such a conjecture.

We have studied the enhancement of heat transfer in gases by inhomogeneous electric fields in some detail[4]. Our results show that this effect, which, for convenience we call electrocooling, is simply an ionic drag phenomenon (that is, the electric wind of Chattock[5]) in which the convective heat transfer coefficient is proportional to the fourth root of the induced corona current. We derived a theoretical expression which predicts such corona-induced heat transfer within 20% of experimental values for electronegative gases such as air. As a particular type of forced convection, electrocooling may thus be directly compared to conventional devices such as air jets or blowers.

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Scrotal asymmetry and Rodin's dyslexia

MCMANUS[1] has pointed out that most artists depict the left testicle of man lower than the right, in accordance with the facts[2]. An exception occurs in Rodin's famous sculpture *L'Age d'Airan* where the right testicle of the figure seems (from photographs we have seen) to hang lower than the left. We should like to suggest that Rodin was genuinely confused about left and right. Evidence that Rodin suffered from a specific reading disability, which is commonly associated with left–right confusions[3], has been reviewed by Thompson[4]. So badly did the artist do in school during his early years that his father was informed: "He is ineducable. The sooner you put him out to work, the better. But I doubt if he can ever make a living."[5].

Some of the facts about scrotal