100 YEARS AGO

Readers of prospectuses of educational institutions and polytechnics may have noticed that of late years there has been a tendency to convert the teachers into professors. The nature of the institution in which the instructors can rightly use the latter title is apparently a matter of opinion, and it is becoming worth while to define the duties and position of a professor. Miss Catherine Dodd describes in the *National Review* how she asked 105 primary school children, between the ages of ten and fourteen, to give this definition, among others. Here are some of the attempts: – "A man who has passed a high examination." "A very clever man." "One who can do his work easily." "A man skilled in sense." That a professor has a certain social standing is evident from the definitions which describe him as "a man who is well off," and "a man who lives in a nice house." Among the vague definitions are the following: – "A person who professes to do something." "A man who says he can do anything." But the children's general idea is that a professor teaches music, dancing, or languages, or performs conjuring tricks. Thus, "A professor teaches all kinds of instruments." "He is a gentlemen that generally plays at balls," and "a man who knows clever tricks." To correctly define a professor would probably prove a difficulty to many children of older growth.

From *Nature* 8 September 1898.

50 YEARS AGO

In the editorial article in *Nature* of August 14, in criticizing a recent statement of the Atomic Scientists' Association, it is argued that collaboration between scientific men east and west of the 'Iron Curtain' may be undesirable, because it is likely "to promote, for the present, a one-way traffic to the disadvantage of the Western democracies" … It is stated in the editorial that the man of science in totalitarian countries is essentially a servant of the State, and that it is treason for him to divulge any knowledge save as the State allows. But in fact this statement is true only in the opinions of the men who control the Government of the U.S.S.R.; we may be sure that most scientific men in the satellite countries would not take that view of their functions. – N. F. Mott.

From *Nature* 11 September 1948.

in body size[6] — a relationship that is known as 'energy equivalence' because the –3/4 exponent indicates that population energy-use per unit area is independent of body mass.

Enquist *et al.* have shown that plant species of all sizes can achieve the same rates of local resource use. The energy equivalence relationship may thus be one of the most widespread of ecological regularities, yet we have no satisfactory explanation of its origin and maintenance. What prevents evolution from producing species that routinely violate it? Now that the relationship has been identified in plants as well as animals, the answer becomes that much more interesting. It also suggests that the explanation (if there is a single one) must be very general. □

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Mathematics

Nonlinear modes of vibration

Ivar Ekeland

One of the most fundamental results in classical mechanics is that linear systems with n degrees of freedom have n fundamental modes of vibration, and that any motion of the systems can be obtained as a linear combination of these fundamental modes. This combination principle does not hold for nonlinear systems, but one suspects that a few periodic solutions will play a part akin to the fundamental modes. In one case, that suspicion has now been confirmed: H. Hofer, K. Wysocki and E. Zehnder[1] have proved that so-called convex systems with two degrees of freedom must have either two periodic orbits for a given energy, or infinitely many — they cannot have three, or twenty thousand, or a billion.

To appreciate the meaning of this result, begin by considering linear systems with two degrees of freedom, such as the weight fixed to springs in Fig. 1. They can always be described in terms of two harmonic oscillators, with periods T_1 and T_2. If the two periods have a common multiple, that is, if $aT_1 = bT_2 = T$ for some pair of integers a and b, then every motion is periodic, with period T. But if there is no common period then there are exactly two periodic motions, with periods T_1 and T_2, in which one of the oscillators is active and the other is shut down. All other motions arise from superimposing both oscillators, and give rise to the well-known Lissajous curves, which are aperiodic.

You should try to visualize this in its four-dimensional phase space (p_1, p_2, q_1, q_2), where p is momentum and q is position. The potential energy of the system is $V = 2\pi^2(q_1^2/T_1^2 + q_2^2/T_2^2)$, and its total energy, or Hamiltonian, is $H = 1/2(p_1^2 + p_2^2) + V$. The surfaces of constant energy are three-dimensional ellipsoids, around which wind the trajectories of the motion. If T_1/T_2 is rational, all these trajectories are closed, corresponding to periodic motions. If T_1/T_2 is irrational, then there are only two closed trajectories for any energy, corresponding to the two fundamental modes of vibration; all other trajectories are open, filling the whole constant-energy surface. This picture extends simply to any number of degrees of freedom.

But how does it extend to nonlinear systems[2]? Consider a convex potential V defined in n-dimensional Euclidean (flat) space, such that V goes to infinity at infinity. In phase space, the Hamiltonian is still $H = 1/2(p_1^2 + p_2^2) + V$, and the constant-energy surfaces are convex and bounded. The corresponding mechanical system is a

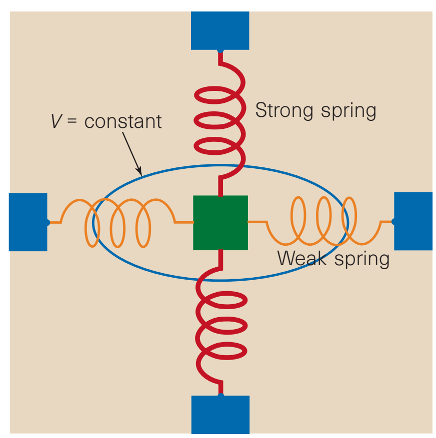

Figure 1 **A dynamical system with two degrees of freedom. A mass is held by springs; strong ones in one direction, weak in the other. It can oscillate up and down rapidly, or side to side slowly, or in more complicated two-dimensional patterns. If the force in the springs depends linearly on their displacement, then lines of constant potential energy V are ellipses, and all the trajectories are well understood. The nonlinear problem is more difficult, but it has now been proved for a large class of systems that there must either be two periodic trajectories, or an infinity.**

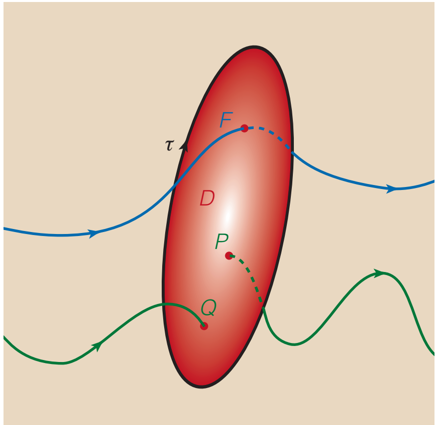

Figure 2 **Trajectories in phase space. For a convex nonlinear system with two degrees of freedom, all trajectories pass through a disk, *D*. The trajectory leaving point *F* returns to the same point (perhaps after crossing the disk several times), so it is periodic; but the one leaving point *P* returns to *Q*, and will continue to meander through the allowed region of phase space aperiodically.**

system of *n* linked nonlinear springs.

It has already been proved that, for nonlinear systems of this kind, every energy has at least two periodic solutions. Drawing a parallel with the linear case, we might guess that a nonlinear system with *n* degrees of freedom would carry at least *n* periodic solutions. That is unproved, but might well be true; it is what mathematicians call a conjecture.

This conjecture raises a question about the global behaviour of nonlinear systems. Much of what is known about nonlinear systems arises from perturbation theory, and gives results valid only for small values of some parameter. There is nothing of the kind here — instead we are asking whether we can build some specific nonlinear system that has only so many periodic solutions; in other words, we have to understand the dynamics not locally, but on the whole of the energy level. Unfortunately, systems whose trajectories can be written explicitly in terms of the initial data and elapsed time are rare. So to prove results about general nonlinear systems one cannot make explicit computations but must resort to other, more geometrical, methods.

As I mentioned before, fixing the energy level to some constant yields a three-dimensional hypersurface that is followed by any trajectory. Hofer *et al.*[1] then find, by variational methods, the periodic trajectory τ that is in some sense the simplest (in the linear case, it would be the one with the shortest period). Then they show using the theory of partial differential equations that τ defines the boundary of a two-dimensional disk *D* through which *all* trajectories must pass (Fig. 2). If, for instance, *P* is a point on the disk *D* itself, the trajectory issuing from *P* will hit *D* at some other point *Q*, thereby defining a map from *D* into itself. This map turns out to be area-preserving, and it is known that such a map must have either one fixed point (corresponding to one more periodic orbit of the flow) or infinitely many.

Clearly, the proof does not carry over to higher dimensions: if there are more than two degrees of freedom, then the energy level will be five-dimensional or more; and although we could still find a periodic trajectory with much the same variational properties as τ, the two-dimensional disk spanning this closed orbit will not catch all the remaining trajectories because they have too many dimensions to move in. So the corresponding conjecture, that every convex Hamiltonian system with *n* degrees of freedom must carry *n* periodic orbits or infinitely many, remains open.

To show again that all this is far from obvious, let us ask another question: what systems have *n* periodic orbits only? One example is the linear case of harmonic oscillators with no common period. Is there anything else? Michael Herman (personal communication) has shown that there is: for any *n*, there exists a convex system with *n* degrees of freedom that has exactly *n* periodic orbits on a prescribed energy level, and which cannot be decomposed into *n* independent harmonic oscillators by any change of variables. In spite of their nonlinearity, general convex Hamiltonian systems still preserve some of the features of harmonic oscillators. □

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HIV

Setting death in motion

Jean Claude Ameisen

Cell motion, an essential component of embryonic development, continues to be important throughout life. The immune system constantly patrols our bodies, checking for infectious pathogens then regrouping cells to attack at sites of invasion. This complex choreography is controlled by a range of chemokines (and chemokine receptors), which induce and guide the movement of cells along chemokine concentration gradients[1]. But, as reported by Herbein *et al.*[2] (on page 189 of this issue) and Hesselgesser *et al.*[3] (in *Current Biology*), chemokines may also act as death signals. And subversion of such signals by the human immunodeficiency virus (HIV) may participate in the development of disease.

An intriguing feature of HIV pathogenesis is this. How does the virus induce programmed cell death (apoptosis) not only in the population of T cells that bears the CD4 receptor (which is required for viral entry), but also in other cell types? For example, HIV causes the death of $CD8^+$ antiviral effector T cells in the immune system, and neurons in the brain, leading to immune exhaustion and dementia, respectively[4].

We know that infection by HIV-1 requires binding of its surface envelope protein, Env, to both the CD4 receptor and a chemokine receptor[1]. Strains of HIV-1 that are prevalent during the first years of asymptomatic infection bind to members of the CCR chemokine-receptor family, whereas strains that become prevalent at the onset of disease bind the CXCR4 chemokine receptor. Thus, chemokines may have a beneficial effect by acting as natural antagonists of HIV infection in $CD4^+$ cells. But, unlike CD4 molecules, chemokine receptors are widely expressed in both the immune system and the brain.

Herbein *et al.*[2] and Hesselgesser *et al.*[3] now show that the Env protein from X4 strains of HIV-1 (that bind CXCR4), and the natural CXCR4 ligand stromal-derived factor-1 (SDF-1), can induce apoptosis *in vitro* in $CD8^+$ T cells[2] and in a neuronal cell line[3]. Moreover, Herbein *et al.* reveal that the death pathway downstream of CXCR4 signalling in $CD8^+$ T cells is a complex and dynamic process. It involves crosstalk with another population of immune cells, the macrophages, and operates through the tumour-necrosis factor-α (TNF-α)/TNF-receptor II (TNFRII) death-transducing pathway (Fig. 1, overleaf).

Signalling by CXCR4 induces the surface expression of TNF-α in macrophages, and TNFRII in $CD8^+$ T cells. Subsequent contact between the macrophages and T cells then triggers T-cell death. Although Herbein *et al.* detected only a low percentage of apoptotic $CD8^+$ T cells, cell loss was high — most of the apoptotic corpses were probably ingested by macrophages, rapidly clearing the scene[5]. Interestingly, this Env-induced death of $CD8^+$ T cells through CXCR4 and then TNF-α/TNFRII signalling, resembles the Env-induced death of $CD4^+$ T cells through CD4 and then Fas ligand (FasL)/Fas receptor signalling[6]. Death of $CD4^+$ T cells also involves the (optional) recruitment of macrophages[7]. Moreover, TNF-α/TNFRII and FasL/Fas belong to the same family of death-transducing ligand/receptor pairs,