matters arising

Lunar magnetism

In a recent series of papers, Runcorn[1,2] has maintained that the observation of a very small lunar surface dipole field ($\lesssim 0.05\gamma$) implies that there used to be a fairly strong interior lunar dipole moment ($\gtrsim 3.2\times10^9$ yr ago). He contends that if such a field had disappeared in the past 3.2×10^9 yr, the exterior field of the Moon would now be zero. This, he argues, is a direct result from potential theory.

I show, for a very simple model of the Moon, that if a primordial core magnetic field existed, it would give rise to a present day non-zero dipole external field.

Consider a uniformly magnetised core of radius a, embedded in a permeable mantle with outer radius b. The core magnetisation is $M_0 = M_0\hat{e}_3$ and the scalar potential of the magnetic field, H, satisfies $H = -\nabla\varphi(x)$, with $B \sim \mu H$ in the mantle. The boundary value problem is easily solved with the result.

$$\Phi_C(x) = \alpha r \cos\theta \tag{1}$$

$$\Phi_M(x) = (\beta r + \gamma/r^2)\cos\theta \tag{2}$$

$$\Phi_V(x) = \delta/r^2 \tag{3}$$

where C, M, and V refer to core, mantle, and vacuum, respectively; and

$$\begin{aligned}\alpha &= \beta+\gamma/a^3\\ \beta &= -2(1-\mu)A\\ \gamma &= b^3(\mu+2)A\\ \delta &= 3\mu b^3 A\\ A &= 4\pi M_0 a^3/D\\ D &= (2\mu+1)(\mu+2)b^3-2a^3(1-\mu)^2\end{aligned} \tag{4}$$

Now imagine that the magnetising currents in the core die out.

The magnetisation of the mantle in the absence of a core field is then

$$M(x) = \frac{(\mu-1)}{4\pi}\left[-\beta+\frac{2\gamma}{r^3}\right]\cos\theta\hat{e}_r + \left[\beta+\frac{\gamma}{r^3}\right]\sin\theta\hat{e}_\theta \tag{5}$$

The scalar potential, $\Psi(x)$, of the resulting field is

$$\Psi_C(x) = -\frac{2}{3}\,\frac{(\mu-1)}{a^3}(1-a^3/b^3)\times \gamma r\cos\theta \tag{6}$$

$$\Psi_M(x) = \frac{(\mu-1)}{3}[\beta(a^3-r^3) - 2\gamma(1-r^3/b^3)]\,\frac{\cos\theta}{r^2} \tag{7}$$

$$\Psi_V(x) = \frac{(\mu-1)}{3}(b^3-a^3)\beta\,\frac{\cos\theta}{r^2} \tag{8}$$

Equation (8) leads to a non-zero external dipole field. Runcorn's conclusion[1,2], that the external field is zero, is based on his assertion that the potential of the magnetising field has the form of equation (2), but with $\beta = 0$. Clearly, if $\beta = 0$ in equation (2), then $\Psi_V(x) = 0$. It must be emphasised that, using an internal magnetising field, the external field is in general non-zero after the core field has decayed to zero. The solution given by equations (6)–(8) is, in fact, a linear combination of the 'interior' and 'exterior' solutions discussed by Runcorn[2].

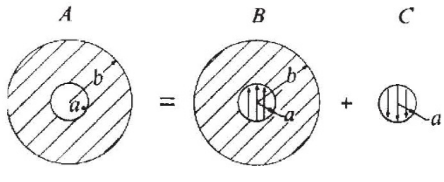

Fig. 1 The superposition of the magnetic fields. The field in *A* where there is no core magnetisation can be derived by subtracting the field of a uniformly magnetised sphere, radius *a*, (*C*) from that of the large uniformly magnetised sphere (*B*). See the equations for details.

The conclusion that the external field is not zero also follows from the linearity of the magnetostatic field equations. The solution of the problem with zero core field can be obtained from the solution with non-zero field, equations (1)–(4), by adding to the fields derived from equations (1)–(4), the field of a uniformly magnetised sphere of radius a, with magnetisation $M' = -M_0\hat{e}_3$. This is indicated schematically in Fig. 1. The resulting external field will clearly be a dipole of reduced strength. The fields resulting from such a superposition are in fact identical to those resulting from equations (7) and (8).

Stephenson *et al.*[3] note that Runcorn's result is strictly true only if the magnetic susceptibility of the mantle is very small, $\sim 10^{-4}$. Although it is clear that the exterior field is of higher order in $(1-\mu)$ than the fields in the other two regions, one must be cautious about arguing that it is therefore negligible. The permeability has been treated as though it were paramagnetic in this simple derivation, but it must, of necessity, be ferromagnetic. To my knowledge the ferromagnetic permeability of the Moon is not known. Dyal *et al.*[4] have, however, found a paramagnetic permeability $\mu \sim 1.01$, that already is larger than the value of $1+10^{-4}$ used by Stephenson *et al.*[3]. This larger value for μ implies the existence of ferromagnetic material[4], the properties of which are undetermined.

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1 Runcorn, S. K., *Nature*, **253**, 701 (1975).
2 Runcorn, S. K., *Phys. Earth planet. Interiors* (in the press).
3 Stephenson, A., Runcorn, S. K., and Collinson, D. W., *Proc. Sixth Lunar Sci. Conf.* (Pergamon, New York, in the press).
4 Dyal, P., Parkin, C. W., and Daily, W. D., *Proc. Sixth Lunar Sci. Conf.* (Pergamon, New York, in the press).

Runcorn replies—The theorem[1,2] which I proved is exactly correct as I stated it: that is, if a spherical shell of any thickness acquires permanent magnetisation, the intensity of which is proportional and parallel to a magnetising field of internal origin, which later disappears, its external field is zero. In the simpler proof[1], the magnetising field was assumed to be that