

ment of accelerated motion requires only the use of the Doppler effect and is merely equivalent to the statement of conservation of energy⁵. Only in special relativity can one equate a Doppler shift with clock retardation which is necessary so that all inertial observers measure the same value for the velocity of light. The principle of equivalence extends this to all freely falling observers.

Furthermore, it can be argued that the full principle of equivalence would be inconceivable before the advent of special relativity: as long as one believed that the velocity of light depends on the motion of the source and/or of the observer, how could one equate an accelerated system with one at rest but in a gravitational field?

We agree with Bishop and Landsberg that relationships between inertial and non-inertial frames are not suitable for discussing the gravitational redshift, and, in fact, this lies at the basis of our treatment in ref. 3.

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¹ Bishop, N. T., and Landsberg, P. T., *Nature*, 252, 459 (1974).

² Kilmister, C. W., *Nature*, 252, 439 (1974).

³ Marsh, G. E., and Nissim-Sabat, C., *Am. J. Phys.*, 43, 266-267 (1975).

⁴ Schild, A., *The Monist*, 47, 20 (1962).

⁵ Misner, C. W., Thorne, K. S., and Wheeler, J. A., *Gravitation*, 187 (Freeman, San Francisco, 1973).

As one of the authors referred to by Bishop and Landsberg¹, I would like to comment before students abandon their text books. Bishop and Landsberg¹ recommend that one should not use Newtonian mechanics with the principle of equivalence, to develop the gravitational redshift. They say that the result "is an absurd conclusion, since Newtonian physics operates with an absolute time". The principle of equivalence also leads to the result that the path of light is curved in a gravitational field. Thus, in another context, it is equally absurd to use the principle of equivalence with special relativity, since light signals are used to synchronise clocks and mark out straight lines in special relativity. Thus, in the presence of gravitational fields, strictly one should only use the general theory. What is done in elementary text books is to add the principle of equivalence on to either Newtonian mechanics or special relativity, whichever is the more convenient. It is an extension in both cases. It serves only as an introduction to the general theory.

A typical approach², considers a light source and detector at the rear and front ends respectively of an accelerating spaceship. The spaceship is considered from

the inertial reference frame in which it is instantaneously at rest, when the light is emitted. Both source and detector have an acceleration a in this frame. The proper length l of the spaceship can be measured using a ruler at this instant. The signal takes a time $t \approx l/c$ to reach the detector, by which time the detector has a speed $v \approx at \approx al/c$ relative to the inertial frame. Provided $v \approx al/c \ll c$, the first order Doppler effect, given by the wave theory of light, is adequate, so that the fractional change in frequency $\Delta v/v \approx v/c \approx al/c^2$.

According to the principle of equivalence, the difference in frequency should be the same, if the spaceship were at rest in a gravitational field of intensity $-a$. Provided $al/c^2 \ll 1$, it is an unnecessary over-elaboration to use the equations of hyperbolic motion, and in a first introductory course the use of Newtonian mechanics and the first order Doppler effect, based on the wave theory of light, is far more convenient. In practical cases $\Delta v/v \sim 2 \times 10^{-15}$ in the experiments of Pound and Rebka and $\sim 2 \times 10^{-6}$ for light from the Sun, so that $al/c^2 \ll 1$, and the simple theory is accurate enough. Near black holes one would have to use the general theory of relativity. This approach, based on the principle of equivalence, should only be treated as a teaching aid, to give a first insight into some of the modifications of both Newtonian mechanics and special relativity required by the general theory of relativity.

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¹ Bishop, N. T., and Landsberg, P. T., *Nature*, 252, 459 (1974).

² Rosser, W. G. V., *Introductory Relativity*, 264 (Butterworth, London, 1967).

³ Pound, R. V., and Rebka, G. A., *Phys. Rev. Lett.*, 4, 337 (1960).

BISHOP AND LANDSBERG REPLY—We shall explain the points raised by referring to the paragraphs in ref. 1 as $a-e$, those in ref. 2 as $a-c$ and those in ref. 3 as $a-g$. We referred^{3b} to a non-Minkowskian metric. This does not imply that we considered the uniform acceleration to produce space-time curvature^{1a}. We had in mind flat (that is, $R_{\beta\gamma\delta}{}^\alpha \equiv 0$) non-Minkowskian metrics of the type⁴

$$ds^2 = g_{44}(x)c^2 dt^2 - \frac{c^4}{4\kappa^2 g_{44}} \left(\frac{dg_{44}}{dx} \right)^2 dx^2 - dy^2 - dz^2$$

We thus agree with Marsh and Nissim-Sabat^{1a} that the existence of a gravitational redshift does not imply space-time curvature, but we do not accept that any

such curvature is suggested by us^{3b}. Rosser^{2a} regards curved light ray paths, and therefore accelerated frames, as unacceptable in special relativity. We^{3d} take the opposite view, which is implicitly supported by Marsh and Nissim-Sabat^{1a}.

For the Newtonian case we used a particle picture of light^{3d} in which light particles are emitted with a constant speed relative to the source, and move according to Newton's laws of particle mechanics. The equality of inertial and gravitational mass leads to the equivalence principle for particles, and thus also for Newtonian optics. From our point of view, therefore, the equivalence principle is completely contained in Newtonian physics, as opposed to special relativity where the principle has to be grafted on, since this theory is not about gravity. Our correspondents disagree with this view (and with each other), the view of Marsh and Nissim-Sabat^{1b} being closer to us than that of Rosser^{2a}. For a wave theory of light (advocated in ref. 2c) the velocity of light is constant with respect to an ether and the equivalence principle does not then apply to Newtonian optics, as also remarked by Marsh and Nissim-Sabat^{1a}. The answer to the question raised there is: "By using a particle picture of light".

An important point of our letter was the view reiterated by Marsh and Nissim-Sabat^{1e} but not taken account of by Rosser^{2b}: the replacement of a uniform gravitational field by a uniform acceleration for source and detector is always magically transformed to a Doppler effect argument for an accelerated detector, thus neglecting the motion of the source. This is a non-trivial step since two times are needed at the source if a frequency is to be defined there. Thus the switch from one model to the other has to be justified, and if presented clearly to beginners^{2c} ought to elicit from them a request for a proof. Our work⁴ may be regarded as providing such a proof for special relativity and, in so far as this had never been done in the past, one was faced by a misuse of the equivalence principle. For Newtonian physics we show⁴ that it is this magical switch in the usual argument that leads apparently to the absurdity of clock retardation by way of the Newtonian Doppler effect. As developed by us⁴, one finds a zero Newtonian gravitational redshift but a non-zero Newtonian Doppler effect for an accelerated detector, and the usual equations for special relativity. These results seem to us clear and consistent.

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¹ Marsh, G. E., and Nissim-Sabat, C., *Nature*, 257, 517-518 (1975).

² Rosser, W. G. V., *Nature*, 257, 518 (1975).

³ Bishop, N. T., and Landsberg, P. T., *Nature*, 252, 459 (1974).

⁴ Landsberg, P. T., and Bishop, N. T., *Foundations of Physics* (in the press).