matters arising

Chandler wobble and viscosity in the Earth's core

VERHOOGEN[1] has recently revived viscous dissipation at the core–mantle boundary as a possible energy sink for the Chandler wobble. His result seems to be based on a misunderstanding of the work of Jeffreys[2] (page 257) and Munk and MacDonald[3] (page 170). He correctly points out that Jeffrey's equation (13) requires multiplication by a factor $4\pi/\omega$, where ω is the diurnal frequency. In fact it contains a further error, in that $(l_1^2+m_1^2)$ should be replaced by $(\dot{l}_1^2+\dot{m}_1^2)/\omega^2$. This is true in the fifth (1970) edition of *The Earth*, as well as in the fourth, cited by Verhoogen. Contrary to Verhoogen's statement, however, this error is not carried into the numerical calculation leading to Jeffreys' equation (14).

In agreement with other authors, Verhoogen takes the viscous dissipation per unit area at the core–mantle interface to be of the order $\rho V^2(\nu\omega)^{1/2}$, where ρ is the density and ν is the kinematic viscosity of the core just below that boundary, and V is the velocity of the core relative to the mantle in the Chandler wobble mode. Let γ be the angle between the instantaneous axis of rotation of the mantle and the angular velocity (more properly, the total vorticity vector) of the liquid core, and let α be the amplitude of wobble of the mantle. Then $V=\gamma a\omega$, where a is the radius of the core–mantle boundary. In Jeffreys' notation $\gamma=(\dot{l}_1^2+\dot{m}_1^2)^{1/2}$ and $\alpha=(l^2+m^2)^{1/2}$. Verhoogen errs in assuming that γ can be identified with α.

In fact, as Jeffreys and Vicente[4] showed, the liquid core hardly participates at all in wobble in the Chandler mode, that is, its vorticity vector remains nearly aligned with **H**, the invariable total angular momentum of the wobbling Earth. Clearly γ is the amplitude of the nutation in space accompanying the Chandler wobble. If, as Verhoogen suggests, γ is of the same order as α ($\simeq 0''.14$), this nutation could hardly have escaped detection by astronomers! Rochester *et al.*[5], in their equations (10) and (11), show explicitly that

$$\gamma = |\omega\times\mathbf{H}|/|\omega||\mathbf{H}| \simeq \alpha\sigma/\omega$$

where $\sigma=\omega/435$ is the frequency of the Chandler wobble. (This relationship between γ and α was implicitly taken into account by Jeffreys in the fourth and fifth editions of *The Earth*, to correct his equations (13) and (14) as they were given in the third (1952) edition.) It follows that $V=\alpha a\sigma$, in agreement with Munk and MacDonald. Then the rate of dissipation of wobble energy by viscous core–mantle coupling is

$$F=4\pi\rho a^4\sigma^2\alpha^2(\nu\omega)^{1/2}$$

This differs by a factor $(\sigma/\omega)^2$ from Verhoogen's equation (1), and leads to Jeffreys' equation (14) when the appropriate numerical values are substituted.

Verhoogen's estimate of the viscosity required to damp the Chandler wobble is therefore too small by a factor $(\omega/\sigma)^4$, that is, viscosities of the order of those calculated by Gans[6] and Leppaluoto[7] ($\sim 10^{-5}$ to 10^{-6} m^2 s^{-1}) fall short of what is needed to damp the wobble by a factor of order 10^{10}. The situation is still as it was left by Jeffreys and by Munk and MacDonald, and we must look elsewhere for the energy sink for the Chandler wobble.

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1 Verhoogen, J., *Nature*, **249**, 334–335 (1974).
2 Jeffreys, H., *The Earth*, fourth ed. (Cambridge University Press, Cambridge, 1959).
3 Munk, W. H., and MacDonald, G. J. F., *The Rotation of the Earth* (Cambridge University Press, Cambridge, 1960).
4 Jeffreys, H., and Vicente, R. O., *Mon. Not. R. astr. Soc.*, **117**, 142–161 (1957).
5 Rochester, M. G., Jensen, O. G., and Smylie, D. E., *Geophys. J. R. astr. Soc.*, **38**, 349–363 (1974).
6 Gans, R. F., *J. geophys. Res.*, **77**, 360–366 (1972).
7 Leppaluoto, D., thesis, Univ. California (1972).

VERHOOGEN[1] has attempted to revive the idea that the viscosity of the core can account for the observed damping of the Chandler wobble. There is, however, an error of principle in Verhoogen's estimation of relaxation time, τ (his equation (3)): the angle α between the axes of rotation of core and mantle has been erroneously assumed to be equal to the angular amplitude, α_w, of the Chandler wobble.

As the core remains approximately motionless, that is, it does not participate in the Chandler wobble[2], the direction of the rotation axis of the core almost coincides with that of the Earth's total angular momentum vector, **H**. So α is nearly equal to the angle between the rotation axis of the mantle and **H** (the 'sway' amplitude according to Munk and MacDonald's[3] terminology), which is known to be $[\sigma/(\sigma-\omega)]\alpha_w \approx (\sigma/\omega)\,\alpha_w$. Here, ω and σ are the diurnal and Chandler frequencies, respectively. If a is the core radius, the differential velocity between the core and mantle is therefore given by $V=\alpha a\omega=\alpha_w a\sigma$, which is just in agreement with Munk and MacDonald's formula. The wobble energy, E_w, of the mantle is given by

$$E_w=(1/2)A_m[(C_m-A_m)/C_m]\,\omega^2\alpha_w^2$$

Thus, Verhoogen's expression for τ turns out to be in error by a factor of $(\omega/\sigma)^2=(430)^2$.

Using the most recent and accurate determination of polar motion[4], one of us (Y.S.Y.) has found that the value of the damping factor, Q, lies between 40 and 60. The corresponding limits on τ are 15 and 23 yr. Assuming that $\tau=20$ yr, we obtain an estimated value for the kinematic viscosity of the core of about 5×10^8 cm^2 s^{-1}. Such a value is regarded by most geophysicists as extremely high[5]. It has to be concluded, therefore, that damping of the Chandler wobble is unlikely to be attributable to core viscosity.

Note added in proof: Verhoogen's and our considerations do not depend on the excitation mechanism of the Chandler wobble and have only a kinematic character. One can consider the value $\alpha_w a$ as the amplitude of the displacement of the mantle relative to the core in the course of the Chandler period σ^{-1}. For this reason the differential velocity

$$V\sim\alpha_w a\sigma$$

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1 Verhoogen, J., *Nature*, **249**, 334–335 (1974).
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3 Munk, W. H., and MacDonald, G. J. F., *The Rotation of the Earth* (Cambridge University Press, Cambridge, 1960).
4 Fedorov, E. P., *et al.*, *Motion of the Earth's Pole from 1890 to 1969* (Naukova Dumka, Kiev, 1972).
5 Rochester, M. G., in *Earthquake Displacement Fields and the Rotation of the Earth* (edit. by Manshinha, L., Smylie, D. E., and Beck, A. E.), 136 (Reidel, Dordrecht, 1970).

VERHOOGEN REPLIES—Viscous damping of the Chandler wobble depends on two angles. The first, α_w, is the angle between e_3 (a unit vector along the mantle's principal axis of inertia) and ω_m, the angular velocity of the mantle. The second angle is the angle α_F between ω_m and ω_c, the