Nuclear incompressibility

from P. E. Hodgson

MANY different measurements of the sizes of atomic nuclei have shown that their volumes are closely proportional to the number of nucleons they contain. It is thus a very good approximation to assume that the nuclear radius is proportional to $A^{1/3}$, and indeed the relation $R = 1.25\ A^{1/3} \times 10^{-1/3}$ cm is widely used. This result may also be expressed by saying that nuclear matter is incompressible, so that the addition of more nucleons to a nucleus does not increase its density but only its size.

But nuclear matter is not infinitely incompressible, and several lines of evidence enable us to estimate its compressibility. None of these is very direct, so there is still considerable uncertainty in the values obtained.

In classical physics, incompressibility is measured by the energy needed to reduce the volume (or the density) by a definite amount. Since at equilibrium the rate of change of energy with density is zero it is usual to define the incompressibility K by the second differential evaluated at equilibrium radius R:

$$K = R^2\, d^2\varepsilon/dr^2$$

evaluated at $r = R = r_0 A^{1/3}$, where ε is the binding energy per nucleon.

Using the semi-empirical mass formula and neglecting derivatives of symmetry terms gives

$$K = K_M + K_S A^{-1/3} + 6e^2Z^2/5r_0A^{4/3}$$

where K_M is the nuclear matter incompressibility and K_S the nuclear surface incompressibility. This shows that the nuclear incompressibility is a function of A.

The incompressibility of nuclei can be estimated indirectly in two ways, one depending on our knowledge of the effective nucleon–nucleon forces and the other on the identification of compression oscillations.

One of the most widely used nuclear theories derives from the Hartree self-consistent field theory for atoms. In this theory each electron is treated as moving in the field of the nucleus and of all the other electrons. If a reasonable estimate of this field is made, it can be used in the Schrodinger equation to calculate the wave functions of the electrons, and hence to give a more accurate estimate of their field at any point. This calculation is continued until it converges, so that the field coming from the solution of the Schrodinger equation is the same as the field used to solve the equation.

Such calculations have been made for nuclei, with appropriate modifications to take account of the absence of a central particle, and the equations are usually solved by electronic computers. The complete calculation from the nucleon–nucleon interaction is very complicated, so simpler methods have been developed using effective interactions. One of the most successful of these is the Skyrme interaction, and Vautherin and Brink (*Phys. Rev.*, **C5**, 626; 1972) have used it to calculate a wide range of nuclear properties from the nuclear Hartree equations. The charge and matter distributions of a range of nuclei, together with the energies of their single-particle states are given very well, so it is perhaps reasonable to accept the values of other nuclear properties derived from the same interaction, even though they cannot be verified independently.

Among these properties is the nuclear incompressibility, and the calculations of Vautherin and Brink gave values of 370 and 342 MeV for the nuclear matter incompressibility calculated from two of their most successful interactions. Earlier nuclear matter calculations by Brueckner and Gammel (*Phys. Rev.*, **109**, 1023; 1958) starting from a phenomenological form of the nucleon–nucleon interaction gave K = 187, 167 and 172 MeV for three different interactions, while Campi and Sprung (*Nuclear Physics*, **A194**, 401; 1972) found K_M = 190, 213 and 295 for three more forces. Statistical calculations by Stocker (*Nuclear Physics*, **A166**, 205; 1971) gave K = 180, 230, 195 for three different forces.

Pandharipande (*Phys. Lett.*, **31B**, 635; 1970) has calculated incompressibilities for a range of nuclei using the constrained Thomas–Fermi method and finds values ranging from K = 123 MeV for ^{40}Ca and 176 MeV for ^{120}Sn to 190 MeV for ^{208}Pb. These values are given by the above expression if K_M = 266 MeV and $K_S = -496$ MeV.

These calculations with nuclear forces carefully adjusted to fit certain nuclear properties give values of the nuclear matter incompressibility that vary in the range 150–400 MeV. This indicates that the incompressibility depends on aspects of the nucleon–nucleon interaction that do not strongly affect the more easily measurable properties of nuclei, so it is difficult to know the reliability of the results obtained.

Another approach to the problem of determining the nuclear incompressibility makes use of a particular kind of nuclear oscillation called a 'breathing oscillation'. The more familiar types of nuclear vibrations are shape vibrations, like those of a liquid drop or a jelly, and these can take place in an incompressible medium. But if the medium is compressible, however slightly, it can oscillate simply by changing its size, while remaining all the time spherical. It would look as if it were breathing and hence the name 'breathing mode' for these oscillations.

The higher the incompressibility, the higher the energy of the excited nuclear state corresponding to such an oscillation. Using a hydrodynamical model (Walecka, *Phys. Rev.*, **126**, 653; 1962) the energy is given in terms of the incompressibility by

$$E \approx (\hbar^2\, K/m\, r_{\mathrm{RMS}}{}^2)^{1/2} \approx 6.9\, K^{1/2}\, A^{-1/3}$$

(Pandharipande, *Phys. Lett.*, **31B**, 635; 1970).

It is not easy to identify such states in nuclei. They have a monopole vibrational character and so have spin and parity 0^+ and strengths that exhaust a large fraction of the EO sum rule. Such states should be readily excited by reactions like inelastic electron scattering. One such state was reported by Pittham and colleagues (*Phys. Rev. Lett.*, **33**, 849; 1974) at 8.9 MeV in Pb, but this corresponds to a rather low incompressibility of $K \approx 90$ MeV, so this identification remains uncertain (*Nature*, **253**, 92; 1975).

In oxygen detailed calculations by Sharp and Zamick (*Nuclear Physics*, **A223**, 333; 1974) gave about 28.5 MeV as the energy of the breathing mode state, but it is not yet possible to identify this with an actual state. The difficulty of finding the breathing mode states thus prevents them being used to determine the incompressibility at the present time, but the development of more specific ways of exciting these states may eventually make this possible.

Breakdown of superfluidity

from a Correspondent

WHAT determines the maximum velocity with which superfluid helium will flow without friction? Can we understand the dissipative processes that occur at supercritical velocities? These are fundamental questions that low-temperature physicists have been asking ever since Landau published his classic theory of the mechanism of superfluidity (*Fiz. Zh.*, **5**, 71; 1941 and **11**, 91; 1947). A recent paper by Phillips and McClintock (*Phys. Rev. Lett.*, **33**, 1468; 1974), describing observations of the drag on an ion moving at high speed through helium, shows that we still do not have complete answers.

The liquid phase of the common isotope of helium, ^{4}He, becomes a superfluid when it is cooled below a temperature of about 2 K. The superfluid transition is associated with a form of 'Bose condensation' in the liquid, in which an appreciable fraction of the atoms accumulate in one particular single-particle quantum state (so that all the condensed atoms have, for example, exactly the same momentum). At a finite temperature the superfluid phase behaves like a mixture of two fluids: a 'normal' component, behaving