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- ¹ Brandi, E., and Jaschek, M., *Publ. astr. Soc. Pacif.*, **82**, 847 (1970).
- ² Adelman, S., *Astrophys. J. Suppl. Ser.*, **26**, 226 (1973).
- ³ Zeidel, A. N., Prokof'ev, V. K., Raiskii, S. M., Slavnyi, V. A., and Shreider, E. Ya., *Tables of spectral lines* (Plenum, New York, 1970).
- ⁴ Seeger, P. A., Fowler, W. A., and Clayton, D.D., *Astrophys. J. Suppl. Ser.*, **11**, 121 (1965).
- ⁵ Meggers, W. F., Corliss, C. H., and Scribner, B. F., *United States National Bureau of Standards Monograph 32*, part 1 (US Government Printing Office, Washington, DC, 0000).
- ⁶ Guthrie, B. N. G., *Observatory*, **89**, 224 (1969).
- ⁷ Guthrie, B. N. G., *Astrophys. space Sci.*, **13**, 168 (1971).
- ⁸ Havnes, O., and Conti, P. S., *Astr. Astrophys.*, **14**, 1 (1971).

Maxwellian relative energies and solar neutrinos

THE solar neutrino discrepancy is now regarded^{1,2} as very serious. The general attitude is that the Sun is trying to tell us something but no one is quite sure what. I wish in this note to add to the list of possible *ad hoc* explanations; namely that the scarcity of ⁸B neutrinos reflects a departure of the distribution of relative kinetic energies from the Maxwellian distribution. I have not been able to calculate a cause of the indicated depletion of the high energy tail of relative energies, so this explanation must be limited to the associated effect.

For an ideal gas (which the solar plasma is not) the Maxwellian distributions of single-particle energies can be shown³ to be equivalent to a Maxwellian distribution for the motion of the centre of mass of the summed masses multiplied by a Maxwellian distribution for the relative motion of the reduced mass. Thermonuclear reactions are calculated with the last of these Maxwellian distributions. For a gas having long range interactions, such as the solar plasma, the distribution function cannot strictly be obtained in such a simple way. The particles undergo many-body collisions and the energy assignable to a particular pair of particles is not defined. A powerful theory could calculate a distribution of relative energies, but only by projecting out of the total distribution its two-particle projection. The usual argument is that the average Coulomb energy is smaller than the average thermal energy, so that an ideal gas is a good approximation.

Nonetheless we should perhaps be cautious, because the thermonuclear reactions involve improbable high energy fluctuations. For example, in a recent model⁴ of the solar centre having $T_c = 14.9 \times 10^6$ K the most effective⁵ energy E_0 for the pp reaction is $E_0(\text{pp}) = 4.58 kT$, and the reaction ³He (α, γ) ⁷Be leading to production of ⁸B involves pairs whose average relative energy is $E_0(3, 4) = 17.4 kT$ — very far out on the tail of the distribution. The power for the Sun comes from energies of a few kT , whereas the neutrinos come from nearly $20 kT$; that causes the ⁸B neutrino flux to be strongly modified if the high energy tail of the two-particle distribution is depleted.

To illustrate these effects I have recalculated the Sun assuming all Maxwellian distributions are multiplied by the same factor $\exp f(E)$, where $f(E)$ is a slowly varying function of energy. I find that the ⁸B flux from the Sun depends on d^2f/dE^2 : if $d^2f/dE^2 < 0$, the ⁸B flux is reduced and if $d^2f/dE^2 > 0$ it is increased. The findings are most easily discussed by expanding $f(E)$ in Taylor series as $f(E) = \beta_0 - \beta_1 E - \beta_2 E^2 + \dots$ and ignoring higher terms as having negligible importance near E_0 .

The constant β_0 is set by requiring that the distribution function remain normalised, and is a function of β_1 and β_2 .

The β_1 term has no physical effect although it might seem to have, since it does seem at first glance to reduce the high energy tail. This would be true at fixed T , but to maintain the Sun's luminosity at its known value, its central temperature would have to be increased sufficiently to counteract the effect of β_1 on the solar power. This increase in T raises the ⁸B neutrino flux back to exactly the value it had for $\beta_1 = 0$. Thus β_1 may be ignored as having no effect. For the ⁸B neutrinos to be reduced in a Sun of fixed luminosity, the deviation from Maxwellian must decrease faster than $\exp(-\beta_1 E)$ between $E_0(\text{pp})$ and $E_0(3, 4)$; that is, d^2f/dE^2 must be negative.

Suppose then that $f(E) \approx \beta_0 - \beta_2 E^2$ and imagine that β_2 is a small number so that no great violence is done to the distribution. It is convenient to think of $\beta_2 E^2$ as being $\delta(E/kT)^2$, where $\delta = \beta_2(kT)^2$ is a smallness parameter for the deviation. My calculations show that values in the range $5 \times 10^{-3} < \delta < 10^{-2}$ are adequate to suppress the neutrino flux to Davis's limit¹. Values of δ in this range would mean that the deficiency in the distribution function is only a few per cent at a few kT , and at $E = 10 kT$ the rate of change of the distribution function with energy due to the β_2 term is still only 10% of the Maxwellian rate of change. These values of δ entail negligible corrections to the equation of state but large corrections to the ⁸B neutrino flux.

Using the same⁴ unperturbed model of the solar centre, I find that the ratio of PPI completions to PPII + III is

$$[\text{PPI}/(\text{PPII} + \text{III})] \approx 10.36 \exp(117\delta) - 0.25 \quad (1)$$

and is 10.11 for the unperturbed model. I also calculate that the ⁸B neutrino flux stands in relation to the unperturbed flux as

$$\frac{\Phi_{\nu}({}^8\text{B}, \delta)}{\Phi_{\nu}({}^8\text{B}, \delta=0)} \approx 10^{-100\delta} \quad (2)$$

For the two values $\Phi = 5 \times 10^{-3}$ and 10^{-2} , the ⁸B neutrino flux is reduced by factors of 3 and 10 respectively. The flux of neutrinos from ($e^- + {}^7\text{Be} \rightarrow {}^7\text{Li} + \nu$) is reduced by factors of 1.7 and 3.0 respectively in a straightforward application of equation (1). The ¹³N neutrino flux, which scales as $\exp(-248\delta)$, is reduced by factors 3.5 and 12 respectively. Thus this *ad hoc* suggestion can substantially lower the neutrino detection rate even for sufficiently small value of δ that its value is otherwise almost inconsequential. The only other major consequence that comes readily to mind is that the equilibrium concentration of ³He at the solar centre will have been increased by the factor $\exp(106\delta)$, which is about 2.7 at $\delta = 10^{-2}$.

These suggestions sound implausible because we are led to believe that the Maxwellian must be correct, even in a non-perfect plasma and even at the level of $\exp(-20)$; but the lack of a positive detection of solar neutrinos by Davis¹ is also implausible. I have been unable to treat the difficult aspects of many-body physics involved, so I write in the hope of more authoritative treatment from others.

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¹ Davis, R., jun. and Evans, J. C., *Proc. XIII int. Conf. Cosmic Rays* (Denver 1973).

² Bahcall, J. N., *Astr. J.*, **76**, 283 (1971).

³ Clayton, D. D., *Principles of Stellar Evolution and Nucleosynthesis* chapter 4. (McGraw Hill, New York, 1968).

⁴ Bahcall, J. N., Bahcall, N. A., and Ulrich, R. K., *Astrophys. J.*, **156**, 559 (1969).