Their suggestion that Piaget's techniques did not involve memory is most surprising. Their "rereading" of Piaget's work has not been complete and they have missed a basic distinction made by Piaget himself between experiments of the type which we are criticizing and experiments involving "spontaneous" inferences.

Piaget has done experiments in which children were shown first that $A > B$ or $A = B$, then that $B > C$ or $B = C$ and finally were asked to make an inference about A and C . He has given clear descriptions of such experiments in which the quantities involved were weight⁴, volume⁵ and size^{6,7}. These are the experiments which we are criticizing, as they involve memory as well as inferences.

Youniss and Furth's suggestion that Piaget did not use this technique is, therefore, inaccurate. They wrongly state that his experiments are only of the "spontaneous" variety where the child not only has to make an inference, but also has to provide for himself the information on which the information must be based. We are not criticizing these "spontaneous" experiments and, indeed, I have suggested elsewhere² that the conclusions drawn from them are basically correct.

Finally, a comment on the statement that Piaget's theory "seems fair game to almost any type of reductive analysis based on elementary processes like memory, attention, learning etc." In our paper we were not trying to reduce the theory; we were simply trying to point out that the statement that young children cannot make transitive inferences was based on poorly designed experiments. Youniss and Furth have done nothing to show that the design of the traditional experiments on transitive inferences is any better than we suggest it is. They seem to be saying that the breadth of Piaget's theory excuses him from carrying out properly designed experiments. I cannot agree with this uncritical approach.

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¹ Bryant, P. E., and Trabasso, T., *Nature*, **232**, 456 (1971).

² Bryant, P. E., in *Constraints on Learning* (edit. by Hinde, R., and Hinde J.) (Academic, 1973).

³ Huttenlocher, J., *Psychol. Rev.*, **75**, 550 (1968).

⁴ Piaget, J., and Inhelder, B., *La development des quantites physiques chez l'enfant*, Ch. 10 and 11 (Delachaux and Niestle, Paris, 1941).

⁵ Piaget, J., *The Child's Conception of Time*, Ch. 5 (Routledge and Kegan Paul, London, 1969).

⁶ Piaget, J., *Mechanisms of Perception*, Ch. 7 (Routledge and Kegan Paul, London, 1969).

⁷ Piaget, J., *Genetic Epistemology*, Ch. 3 (Columbia Univ. Press, New York, 1970).

GENERAL

Gamma Distribution in Consumer Purchasing

A LONGSTANDING assumption in the theory of repeat buying has now been explained by results on consumers' brand switching behaviour first published a few years ago¹ together with more recent findings concerning the independence of brand choice and weight of purchasing.

The NBD theory of repeat buying for frequently bought branded consumer goods rests on two assumptions. (i) Purchases of a particular brand by a given consumer in successive equal time-periods are independent and follow a Poisson distribution with constant mean. (ii) The mean rate of purchasing varies from consumer to consumer and follows a gamma distribution in the whole population. One consequence of this model is that the frequency distribution of purchases of a brand for the whole population in any given time period should follow the negative binomial distribution (hence the

name of the NBD model). This and other deductions have generally been found to hold in thousands of cases over the past 15 years²⁻⁴.

There has been considerable evidence supporting the Poisson assumption, and showing that the NBD model is not sensitive to such departures from the Poisson assumption as do occur^{2,5}. There is also overwhelming empirical evidence that different consumers have different mean purchasing rates, but there has been no direct evidence for the assumption that these differences should follow a gamma distribution.

However, a direct explanation for the appearance of the gamma distribution is provided by applying a theorem of Mosimann⁶ to the observed regularities in consumer brand-choice behaviour. This theorem, based on a powerful characterization of the gamma distribution due to Lukacs⁷, is as follows: "Let Y_i ($i=1, 2, \dots, g$) be g non-degenerate and positive random variables, and suppose that they are inde-

pendently distributed. Let $W = \sum_{j=1}^g Y_j$ and define the random

variables P_i ($i=1, 2, \dots, g-1$) by $P_i = Y_i/W$. Then each of the $(g-1)$ random variables, P_i , is distributed independently of W if and only if all Y 's have gamma distributions with the same scale parameter."

Applying this to consumer purchasing for a product-field with g brands, Y_i represents the average rate of buying brand i across different consumers, W represents a consumer's rate of buying the product-field as a whole, and P_i the proportion of a consumer's total purchases which is devoted to brand i .

Now the brand-switching results reported earlier¹ provide evidence that the purchasing rates Y_i are nearly independent, the correlations between them being generally very small. There also is now extensive empirical evidence that the P_i are approximately independent of W ; for instance, the share of purchases devoted to each brand is generally much the same amongst light, medium and heavy buyers of the product-field².

These empirical results together with Mosimann's theorem explain why the variation across consumers of the rate of buying a brand should approximate a gamma distribution. It also provides a link between the theory of repeat-buying of a single item and that of brand choice between different items. The development of a comprehensive model of buyer behaviour embracing single-brand and multi-brand aspects is proceeding.

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¹ Ehrenberg, A. S. C., and Goodhardt, G. J., *Nature*, **220**, 304 (1968).

² Ehrenberg, A. S. C., *Repeat-Buying: Theory and Applications* (North Holland, Amsterdam, 1972).

³ Chatfield, C., Ehrenberg, A. S. C., and Goodhardt, G. J., *J. Roy. Statist. Soc., A*, **129**, 317 (1966).

⁴ Ehrenberg, A. S. C., *Appl. Stat.*, **8**, 26 (1959).

⁵ Chatfield, C., and Goodhardt, G. J., *J. Amer. Stat. Assoc.*, **68**, 344 (1973).

⁶ Mosimann, J. E., *Biometrika*, **49**, 65 (1962).

⁷ Lukacs, E., *Ann. Math. Statist.*, **26**, 319 (1955).

Reference Abbreviations

FROM September 1 it is *Nature's* intention that all abbreviations of references should conform to the style of the *World List of Scientific Periodicals*, fourth ed. (Butterworth, 1963-65). The changeover will be gradual and authors submitting manuscripts from now on are asked to ensure that the references are written appropriately.