

linear wave solutions with sinusoidal profiles and dispersive properties. Going one step further, the non-linear terms may be partially restored by substituting the first approximation to obtain the "Stokes wave" (Lamb, *Hydrodynamics*, art. 250, Cambridge University Press, 1932), which has flatter troughs, and crests of greater curvature, than the sinusoid. This tendency increases as the steepness is allowed to increase until ultimately, Stokes showed, a limiting form for the progressive wave should be obtained, which has a sharp symmetrical wedge-shaped crest enclosing a perfect angle of 120° (maximum slope 30° from the horizontal). Unfortunately, the method of trying to approach the 120° crest by a series of approximations from a sinusoid fails to converge.

Stokes worked in the second half of the nineteenth century; since then a succession of mathematicians have attempted by various means to formulate the full profile of the 120° wave. Basically they all give slightly different results (for the ratio of height to wavelength, for example), and nearly all involve numerical computations which are strenuous even by modern standards. The standing wave presents similar problems and, by contrast, is limited by a 90° crest. It is interesting to observe that although the limiting standing wave has a peak vertical acceleration of $-g$, so that particles at the crest are momentarily "weightless", the corresponding acceleration for the limiting progressive wave nowhere exceeds $-\frac{1}{2}g$.

Longuet-Higgins has derived his formula by the elegant trick of conformally transforming a regular polygon. A hexagon transforms into a wave-like profile with perfect 120° crests; a square yields 90° crests, corresponding to the limiting standing wave. All the hydrodynamical conditions are satisfied except that of constant pressure, but the actual pressure distribution is shown to be within 7 per cent of uniformity. And the profile is so close to the results of laborious numerical approaches as to be nearly indistinguishable on an ordinary graphical plot. Its greatest virtue is its simplicity. With suitable scaling the profile may be written

$$y = \ln \sec x$$

with trough at $x=0$ and crests at $x = \pm\pi/6$ (progressive wave) or $\pm\pi/4$ (standing wave). Simple formulations also exist for the fields of velocity and acceleration. These properties will appeal to marine engineers, who are notoriously sceptical of abstruse mathematical formulae but very concerned with the properties of the steepest waves which are precisely those responsible for the wrecked oil rig or the capsized trawler.

MATERIALS

Anomalies in Perspex

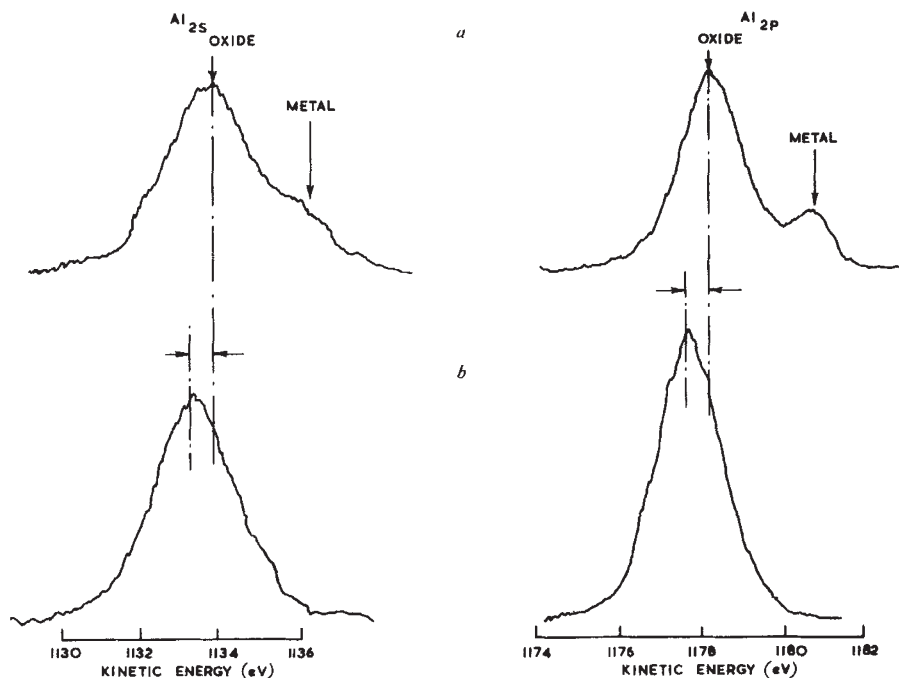
IN the latest of a series of reports about undergraduate projects which have produced worthwhile results, Puttick, of the University of Surrey, describes some curious features of Perspex in *Journal of Physics E: Scientific Instruments* (6, 116; 1973). Much of the experimental work was carried out by Jewers, then a third-year undergraduate at the University of Surrey, under supervision.

The experiment involved a study of how Perspex indents and fractures when a steel ball bearing (3.2 mm) is driven into the surface at speeds of, typically, between 0.017 and 1.7 mm s^{-1} . The work revealed some interesting features

of the way in which cracks develop under these circumstances, but it also brought to light a strange phenomenon that was observed to particular advantage when specimens were unloaded before fracture could occur.

An example is shown in the figure; a spherical region of anomalous refractive index is generated beneath the indentation, and this region defines the inner boundary of the crack, which Puttick likens to a sycamore seed. By applying elementary optics to what amounts to a thick lens embedded in a medium, it turns out that the difference in refractive index is $+1.7 \times 10^{-2}$ and that the change in density in the spherical region, $\delta\rho/\rho$, is 2.9×10^{-2} . By comparison, assuming that all the material that was in the crater (apart

ESCA Probes the Oxide Films on Aluminium



a, Hot-rolled aluminium sheet; b, similar sheet heated in an oxygen-rich flame.

ACCORDING to the literature on the subject, the most recent of which is dated 1954, the thickness of the oxide film ($\gamma\text{Al}_2\text{O}_3$) on aluminium manufactured at temperatures above 400°C can be as large as 2,000 Å. Why then does aluminium produced in this way have a shiny appearance? In next Monday's *Nature Physical Science* (February 26) Tripathi and Clark describe measurements made by ESCA (electron spectroscopy for chemical analysis) which show that the thickness of the film is in fact about 15 Å.

They have used an electron spectrometer to demonstrate that certain photoelectrons released from the metal (as opposed to the oxide) during irradiation with $\text{MgK}\alpha_{1,2}$ X-radiation could not escape if the thickness of the oxide layer

were any more than 15 ± 10 Å. The situation is illustrated in the diagram; the smaller peaks in a are caused by photoelectrons from the metal and the larger ones by electrons from the oxide layer. The spectra in b are for a piece of aluminium sheet that had been exposed to an atmosphere rich in oxygen for 3 min at 600°C . Further oxidation apparently proceeds smoothly.

Clearly the oxide film has increased in thickness to the extent that the photoelectrons from the metal can no longer escape. And Tripathi and Clark also point out that there is a shift in the position of the peak in the two situations, suggesting that when the oxide film is thick there is no longer any electrical contact with the spectrometer so the sample charges up.