14 mission, and in the uplands samples from the Apennine front (Apollo 15). On the basis of this geochemical evidence, we postulate an origin for the highlands by melting and fractionation of at least the outer few hundred km of the Moon.

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Relativistic Precessions of Macroscopic Objects

THE problem of extending the Thomas precession to a macroscopic object spinning about its own axis has recently been considered in connexion with a rapidly rotating disk¹. It was concluded that an arbitrary finite section of the disk would not undergo a Thomas rotation, but instead would experience "Thomas shear stresses" and that only the individual rigid units (point particles) would rotate with the Thomas precession rate. The chief aim of this communication is to point out that, if the gravitational field of the disk is not neglected, this result also holds for the analogous general relativistic precessions, and to re-examine the relativistic precessions from a more realistic microscopic point of view.

If the gravitational field of a spinning disk is not neglected it is evident from the arguments concerning the Thomas rotation of a section of the disk¹ that the above result holds for the analogous general relativistic rotations. If a small section of the disk (near the perimeter, say) was not constrained it would undergo, in addition to the Thomas rotation, two general relativistic rotations given by²

$$\Omega_1 \approx \frac{3}{2} \frac{m}{r^3} (\mathbf{r} \times \mathbf{v}) \quad (1)$$

and

$$\Omega_2 \approx -\frac{I}{r^3} \omega \quad (2)$$

In equations (1) and (2) the units are such that $c = G = 1$; m is the disk mass, $\mathbf{r}$ the radial position vector of the section, $\mathbf{v}$ the tangential velocity of the section, I the moment of inertia of the disk and ω its angular velocity. As the section is constrained

from rotating there will be (from a macroscopic point of view at least) shear stresses introduced that are completely analogous to the Thomas shear stresses. As in the case of the Thomas rotation, only the "point particles" of the disk will rotate.

I shall now consider the relativistic rotations at the point particle level. The results pertain to a macroscopic object in orbit, but they will also be applicable to a section of an object spinning about its axis. In the case of a weakly bound macroscopic object, a sphere of gas, say, in orbit in the gravitational field of a much larger mass (possibly spinning) only the individual point particles in the gas will undergo the relativistic rotations. The sphere of gas as a whole will not rotate because the individual particles are not strongly coupled to one another. But even the strong coupling of the particles is not sufficient to ensure that the entire body will rotate; the coupling forces must also be dependent on direction. For example—assuming the particles to be atoms—if an Na or Cl atom in an NaCl crystal is rotated there will be no torque exerted on neighbouring atoms and hence no torque on the crystal as a whole as the ionic bond is not dependent on direction. This is also true of the metallic bond. Classically, of course, not even an individual atom in an orbiting macroscopic object would be expected to rotate as a whole, for the principal force holding the nucleus and electron together is the Coulomb force, which does not depend on direction. It is not even certain that a nucleus (or for that matter a nucleon) will rotate relativistically as a whole, because it is a composite object. In other words, rotating a nucleon will not necessarily cause the nucleus to rotate, and rotating the nucleus will not necessarily cause the atom to rotate. (Only the electron, a point particle, is known to undergo a relativistic rotation.) Thus a necessary condition for a macroscopic object to undergo a relativistic rotation is that there be strong coupling that is dependent on direction between the composite particles at all levels. It is also apparent that if there is no strong coupling of this kind, no shear stresses will be induced. It follows, for example, that the crust of a rotating neutron star (which is a Coulomb lattice structure) will not experience relativistic shear stresses.

These results are applicable to any macroscopic object including an orbiting (or a laboratory) gyroscope². A gyroscope is merely a convenient means of defining the coordinate axis that is supposed to be rotating with respect to the fixed stars. Physically the coordinate axes may be considered as rigidly attached to the object. That the above conclusions hold for a gyroscope is easily seen by noting that all three of the relativistic rotations are independent of the angular momentum $\mathbf{s}$ of the gyroscope. Hence the behaviour of a gyroscope need only be considered in the limit $|\mathbf{s}| \rightarrow 0$, which is just the case of the macroscopic object discussed above. A metallic gyroscope will not therefore undergo the relativistic precessions. It is not clear, however, whether a gyroscope made of a material involving bonds (say) will precess. The usual result² that $\mathbf{s}$ will precess with respect to the fixed stars, whatever the microscopic structure of the gyroscope, makes the (tacit) assumption that the gyroscope is perfectly rigid and that the microscopic coupling is strongly dependent on direction.

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