These results suggest that multiple criteria are used in selection of simultaneously available inputs. On the basis of these criteria, items may be fed through a single channel, as Broadbent originally proposed.

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Relationship between Stability and Connectedness of Non-linear Systems

Gardner and Ashby[1] have studied the relation between the probability of stability $p(S)$ and the extent of connectedness ("connectance") C of large linear dynamical systems. They found a very interesting on–off-type (numerical) result: for reasonably large systems (number of components $n \geq 10$) there is a critical value, C_c of C, below which the system is almost certainly stable, but above which it becomes almost certainly unstable.

In this communication we outline the effect of non-linearity on this striking $p(S)$–C relationship; the general non-linearity of realistic models of physical systems lends particular importance to such an undertaking.

Let the vector function $x(t) = (x_1(t) \ldots, x(t))$ represent the state of the linear system at time t. The time development of $x(t)$ is given by

$$\dot{x}(t) = Ax(t) \tag{1}$$

that is

$$\dot{x}_i(t) = \sum_{j=1}^{n} a_{ij}x_j(t), \; j=1, \ldots, n \tag{2}$$

The system is assumed to be autonomous, that is, the a_{ij}s are constants independent of t.

The connectance C of the system (1) is defined as the fraction of off-diagonal elements a_{ij} of A that are non-zero. Thus $0 \leq C \leq 1$. For $C=0$ the system is completely disconnected, and (2) separates into n independent, decoupled equations.

The stability (uniform asymptotic stability) S of the linear system (1) can be related to the eigenvalue distribution of A: if all eigenvalues λ_i satisfy $\mathrm{Re}(\lambda_i) < 0$, then (1) is stable, otherwise it is unstable[2].

The $p(S)$–C relationship was discovered in the following manner. (1) A value for n was chosen. (2) The n diagonal elements a_{ii} of A were selected randomly from a uniform distribution in $(-1.0, -0.1)$, that is, the individual x_i were assumed to be intrinsically stable. (3) The x_is were then coupled by generating $n(n-1)C$ non-zero off-diagonal elements randomly from a uniform distribution in $(-1., 1.)$. This completed the construction of A. (4) A was then diagonalized and tested for stability. (5) For a given C steps 2–4 were repeated a large number of times; the fraction of random matrices found stable was then defined as (a measure of) the probability of stability $p(S)$. (6) Steps 1–5 were repeated for different n values.

In general $p(S)$ decreases with increasing C. The larger n, the greater the rate of decrease. The surprising feature is the step-function-like drop of $p(S)$ from 1 to 0 at some critical C value C_c for $n \geq 10$. Gardner and Ashby conjecture that for all large complex dynamical systems such discontinuous behaviour is to be expected.

The influence of non-linearities on the $p(S)$–C relation will be analysed in two stages: (*a*) Effect on S; (*b*) effect on C.

(*a*) Consider the non-linear autonomous system

$$\dot{x}_i = \sum_{j=1}^{n} a_{ij}x_j + Y_i(x_1, \ldots, x_n), \; i=1, \ldots, n \tag{3}$$

where we assume that the Y_is are convergent power series in x_i, beginning with terms of at least the second degree. Then one has the following theorem of Liapunov[3].

(i) If (3) is stable for $Y_i = 0$, all i (that is, if (1) is stable), then the equilibrium ($x_i = 0$, all i) is asymptotically stable whatever the terms Y_i are. (ii) If (1) is unstable (has at least one λ with $\mathrm{Re}(\lambda) > 0$) the equilibrium is unstable whatever the Y_i are. (iii) If (1) has some roots λ with $\mathrm{Re}(\lambda) = 0$ but none with $\mathrm{Re}(\lambda) > 0$, then the terms in Y_i can be chosen such that they have either stability or instability.

This theorem enables us to determine the stability of a class of non-linear systems (3) provided the stability of the precursor linear system (1) is known.

(*b*) In trying to understand the effect of non-linearity on C one is faced with the problem of defining C unambiguously. (This is not trivial. Even in the linear case one makes the tacit assumption that the x_is are physically well defined and meaningful components of the interacting system, that is that the coupling of these components does not destroy their physical identity; otherwise, purely mathematical transformations $x_i' = T(x_i)$ can be constructed that would change C arbitrarily.) We ignore this dilemma and assume that the definition of C used for the linear system is also applicable in the non-linear case. We write

$$Y_i(x_1, \ldots, x_n) = \sum_{j=1}^{n} [\sum_{k=1}^{n} b_{ik}x_k^{\,j}]x_j, \text{ all } i$$

where the b_{ik}s are constant. Then (3) becomes

$$\dot{x}_i = \sum_{j=1}^{n} \{a_{ij} + \sum_{k=1}^{n} b_{ik}x_k^{\,j}\}x_j, \; i=1, \ldots, n \tag{4}$$

Assume that the linear constituent of (4) is diagonal and stable (that is, $C=0$, $a_{ij} = -\delta_{ij}a_{ii}$, $a_{ii} > 0$). Clearly, Liapunov's theorem ensures that by different choices of the b_{ik} one can vary C between 0 and 1, without affecting the stability of (4) ! Consider now a collection of linear systems (generated *via* the Gardner–Ashby procedure) which is characterized by a particular ($p(S)$, C) pair. According to our procedure, we can add non-linear terms to each linear system in the collection without affecting its stability either way ((i) and (ii) of Liapunov's theorem; we assume that the probability of (iii) being operative is negligible). At the same time, we can change the C values at will. In particular, the ($p(S) = 1, C < C_c$) pairs can be changed to ($p(S) = 1, C_c \leq C \leq 1$).

These results indicate that in the presence of non-linearity the concept of critical connectance may be meaningless in its present form. This conclusion does not deny either the existence or the importance of such a concept; it merely indicates that a more fundamental definition of connectance is needed. Perhaps an approach similar to Kauffman's[4] will be more successful.

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