distortions. Is there any reason not to believe in this account of the matter?

Finally, to say that "expectancy" and "contrast" are concepts applicable to illusions ranging from colour changes after wearing red glasses, the "waterfall-effect" of induced movement, the "size-weight" illusion, after-effects of size, as well as the rectilinear projection illusions and presumably also perspective illusions, is either to have a deep insight into the mechanisms of perception or to give such a low resolution description that important differences between them are blurred.

A vital distinction which should not be lost is between (a) neural systems becoming modified, or adapted, by maintained or unusual stimulation ("physiological adaptation"), or by prolonged or unusual sensed object-characteristics ("cognitive adaptation") and (b) normally functioning physiological or cognitive processes, being applied inappropriately in the given situation. Inappropriateness can generate errors and distortions in unusual situations though the system is functioning normally; but to recognize that this is the case we must understand the significance of the physiological and cognitive processes, and discover why they are inappropriately applied, and in what ways they are inappropriate, in the illusion situation. This involves looking at each illusion situation very carefully: facile generalizations prevent our seeing the significance of the phenomena in terms of normal but misapplied, or abnormal, functioning of the perceptual system. Perhaps a theoretical classification is needed here; the following seems useful: (A) physiological illusions, generated by physiological (1) adaptation, (2) inadequacy, (3) inappropriateness; (B) cognitive illusions, generated by cognitive: (1) adaptation, (2) inadequacy, (3) inappropriateness. Although there is much room for debate and further knowledge, all illusions can, I submit, be classified with these headings without doing damage to distinctions we need to bear in mind to understand perceptual effects and use the phenomena as tools for understanding perception.

R. L. GREGORY

*Brain and Perception Laboratory,
Department of Anatomy,
University of Bristol,
University Walk,
Bristol BS8 1TD*

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Model for a Two-adversary Arms Race

A MATHEMATICAL model for a two-adversary arms race which comprises three features, (i) response to adversary's stockpile, (ii) response to his growth rate and (iii) finite resource limitation, can be described by the pair of equations

$$u'_1 = (K_1 u_2 + L_1 u'_2) (a - u_1) \quad (1)$$

$$u'_2 = (K_2 u_1 + L_2 u'_1) (b - u_2) \quad (2)$$

where u_1 and u_2 are the arms stockpile for the two sides; the primed functions, their time derivatives; and a and b , the finite limits to the resources of each. The factors $(a - u_1)$ and $(b - u_2)$ have been used before¹ to describe logistic growth laws, but make the above two equations non-linear.

In this model the responsive coefficients are non-negative constants. If both u_1 and u'_1 are positive, so must u'_2 . Should

one's stockpile be kept stationary ($u'_1 = 0$), the foe's will still grow ($u'_2 > 0$) by the second equation. A slight decrease in one's stockpile (u'_1 negative but small) will not prevent the foe's growth (u'_2 positive by equation (2)) which, in turn, induces a renewed growth in one's stockpile (by equation (1)). Slight rates of disarmament by either side, or both, is not a stable condition. There is an ineluctable return to competitive growth.

There is a critical rate for one's own arms reduction rate, when $K_2 u_1 + L_2 u'_1 = 0$, which induces the foe to keep his stockpile level constant. From this critical condition one's stockpile has an exponential decay with relaxation time L_2/K_2 . Disarming slower than this leads to the above mentioned instability; more rapidly, to the foe's disarming. However, one's disarmament rate must be even more drastic than this to attain stable bilateral disarmament. For, if the adversary's induced disarmament rate is slight, one's own rearmament will be induced. That condition is

$$u'_1 < -K_2 u_1/L_2 - K_1 u_2/L_1 L_2$$

From this, the greater is each stockpile the greater the response to these, and the less the response to the rate of reduction, the more drastic must one's disarmament rate be to induce the foe to disarm rapidly enough to, in turn, assure oneself. By interchanging subscripts the foe's condition is obtained. The solution of these two simultaneous inequalities has two exponential decay terms. For simplicity, if $K_1 = K_2 = K$ and $L_1 = L_2 = L$, these are $\exp -K(1 \pm 1/L)t/L$. The term with the upper sign is the faster and proper decay.

If bilateral disarmament does not obtain, the ensuing arms race can be examined. In the light of the high priority given to a nation's military capability it is reasonable and certainly convenient to neglect the factors $(a - u_1)$ and $(b - u_2)$ in the first equations. These can then be restated as two second order equations

$$(1 - L_1 L_2) u''_1 - (K_1 L_2 + K_2 L_1) u'_1 - K_1 K_2 u_1 = 0$$

and, *mutatis mutandis*, the same for u_2 . Note that the constant coefficients are identical for u_1 and u_2 . If the first coefficient were negative, it would not be possible to have u_1 , u'_1 , and u''_1 all positive or all negative. Since there is no obvious self-correction of this sort in an arms race, $L_1 L_2$ cannot exceed unity.

For the special case $L_1 = L_2 = 0$, there is an exponential growth with argument $(K_1 K_2)^{1/2} t$, and the familiar Lanchester N-square law²

$$u_1^2/K_1 = u_2^2/K_2 + \text{constant}$$

When $K_1 = K_2 = 0$, it follows that $L_1 L_2 = 1$. The two-terms solution of the second order equations is particularly simple if corresponding responsive coefficients are identical, so the subscripts may be omitted.

$$u_1(t) = \frac{1}{2}(u_1(0) \pm u_2(0)) \exp \pm Kt/(1 \mp L)$$

Similarly, for $u_2(t)$. The adversary with the initially smaller stockpile will grow faster until both sides have identical increasing capabilities (provided the finite-resource constraint is not imposed). The growth term increases explosively as L approaches unity. In this model greater response to rate of growth corresponds to more rapid escalation.

AVROM A. BLUMBERG

*DePaul University,
Chicago,
Illinois 60614*

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