1 Bridle, A. H., *Mon. Not. Roy. Astron. Soc.*, **136**, 219 (1967).
2 Yates, K. W., *Austral. J. Phys.*, **21**, 167 (1968).
3 Wall, J. V., Chu, T. Y., and Yen, J. L., *Austral. J. Phys.*, **23**, 45 (1970).
4 Scheuer, P. A. G., and Williams, P. J. S., *Ann. Rev. Astron. Astrophys.*, **6**, 321 (1968).
5 Alexander, J. K., Brown, L. W., Clark, T. A., Stone, R. G., and Weber, R. R., *Astrophys. J. (Lett.)*, **157**, L163 (1969).
6 Alexander, J. K., Brown, L. W., Clark, T. A., and Stone, R. G., *Astron. Astrophys.*, **6**, 476 (1970).
7 Sciama, D. W., *Nature*, **204**, 767 (1964).
8 Bridle, A. H., *Nature*, **219**, 1136 (1968).
9 Reber, G., *J. Franklin Inst.*, **285**, 1 (1968).
10 Shain, C. A., IAU/URSI *Symp. on Radio Astronomy*, 328 (Stanford Univ. Press, 1958).
11 Yates, K. W., and Wielebinski, R., *Austral. J. Phys.*, **19**, 389 (1966).

Centrifugal Force and General Relativity

THIRRING'S solution for a rotating mass shell is frequently used to illustrate the appearance of centrifugal and Coriolis force in general relativity. In the equations of motion of test particles within the shell, terms appear which are of second order in the shell angular velocity, ω. These terms are conventionally identified with "centrifugal force", yet they do bear the relationship to "Coriolis force" that one would expect from Mach's principle. The resolution of this paradox has an important bearing on the foundations of general relativity because, to resolve it, some authors have taken the extreme position that either general relativity or Mach's principle must be abandoned. In this communication we resolve the paradox without abandoning either.

Using the weak field form of Einstein's equations, Thirring[1,2] found the metric associated with a slowly rotating mass shell of mass m, radius r_0 and angular velocity ω. Inside the shell, the geodesic equations contain terms proportional to ω and ω^2. These terms enter the equations of motion in a manner similar to that in which the Coriolis and centrifugal terms enter in Newtonian mechanics; because of this, the ω and ω^2 terms have been interpreted[1-4] as Coriolis and centrifugal force terms. The Coriolis term is equal to that which would be found in a coordinate system rotating with angular velocity

$$\Omega_0 = 4m\omega/(3r_0)$$

Because Ω_0 is proportional to the angular velocity of the shell ω, it has become known as the induced angular velocity of inertial frames. Unfortunately, the ω^2 terms in the equations of motion do not vanish in the same frame as the one in which the ω terms vanish. In Newtonian mechanics, both terms vanish in the same frame, known as an inertial frame. These difficulties led Bass and Pirani[5] to conclude that there was an apparent conflict with Mach's principle (see also ref. 6).

In an attempt to remove some of these difficulties, Bass and Pirani[5] refined Thirring's calculation[1,2] by including contributions from the elastic stress in the shell. Unlike the situation in Newtonian mechanics, the shell elastic stress can contribute to the motion of test particles inside the shell. These authors also raised other questions. For example, they noted that if the density distribution of the shell is given an appropriate dependence on latitude, it is possible to eliminate completely the ω^2 terms from the geodesic equations. Thus even when the contribution from elastic stress is included, an apparent conflict with Mach's principle remains, as was pointed out by Bass and Pirani[5].

Because the geometry of the shell (treated by Thirring[1,2] and Bass and Pirani[5]) remains spherical, some authors have argued that the ω^2 terms cannot represent quadrupole terms. Despite this, we wish to argue that the ω^2 terms are solely quadrupole terms. The quadrupole moment arises as a consequence of the special relativistic mass increase with velocity of the shell. The mass on the equator moves faster than that on the poles. Because the ω^2 terms vanish when the quadrupole moment (in the rest frame of a distant observer) vanishes and are present when the quadrupole moment is present (as can be deduced from the calculations in ref. 5), it is reasonable to conclude that the ω^2 terms are quadrupole terms.

This conclusion can also be reached in another indirect way. The two possible sources of the ω^2 terms are centrifugal and quadrupole (multipole) contributions. The possibility of a centrifugal contribution can be eliminated, however. Because Thirring used the linearized form of Einstein's equations, his results are valid only to first order in m/r_0. Terms of order m^2 and higher are neglected in the linearized (weak field) form of Einstein's equations. On the other hand, using the first order result in the equation as a guide, one would expect the "centrifugal force" terms (of order Ω_0^2) to vary as m^2. Because such terms do not appear in the linearized theory, one cannot correctly interpret the ω^2 terms as "centrifugal force" terms.

Thus the ω^2 terms do not represent "centrifugal forces" in any sense. Thirring's ω^2 terms are due solely to the quadrupole moment associated with the special relativistic mass increase of the shell. Hence there is no conflict with Mach's principle[7,8].

We thank C. Cronström, F. Dyson and J. Weber for reading the manuscript. One of us (J. M. C.) also thanks Dr Carl Kaysen for his hospitality at the Institute for Advanced Study and for support in part by a grant from the Air Force Office of Scientific Research, Office of Aerospace Research.

JEFFREY M. COHEN

Institute for Advanced Study,
Princeton, New Jersey 08540.

WILLIAM J. SARILL

Department of Physics,
Boston University,
Boston, Massachusetts 02215.

Received October 1, 1970.

1 Thirring, H., *Phys. Z.*, **19**, 33 (1918).
2 Thirring, H., *Phys. Z.*, **22**, 29 (1921).
3 Einstein, A., *The Meaning of Relativity*, 99 (Princeton University Press, 1950).
4 Pauli, W., *Theory of Relativity*, 174 (Pergamon Press, 1958).
5 Bass, L., and Pirani, F. A. E., *Phil. Mag.*, **46**, 850 (1956).
6 Hönl, H., and Maue, A.-W., *Z. Phys.*, **144**, 152 (1956).
7 Brill, D. R., and Cohen, J. M., *Phys. Rev.*, **143**, 1011 (1966).
8 Sciama, D. W., *Mon. Not. Roy. Astron. Soc.*, **113**, 34 (1953).

Velocity Measurements in the Deep Western Boundary Current of the South Pacific

IN their schematic theory of the deep ocean circulation, Stommel and Arons[1] require deep western boundary currents in each of the world's oceans as the means by which cold deep water is transported from polar regions to middle and low latitudes. These have been measured directly in the North Atlantic[2], inferred from the density field and property distributions in the South Atlantic[3], and, most recently, detected in similar fashion in the South Pacific, just east of New Zealand and the Kermadec Ridge, during the occupation of transpacific hydrographic sections along latitudes 28° S and 43° S [4]. Subsequently, Reid[5] has made direct current measurements near the bottom in the narrow channel at about 9° S, 169° W, which connects the deep central basins of the North and South Pacific. These demonstrated a northward flow with near-bottom speeds of 5–15 cm/s which is presumably derived from the deep boundary current to the south.

No velocity measurements have been made in the boundary current proper and therefore it was not possible