Stoneley Waves in Anisotropic Media

WAVES at the interface between two isotropic solid half-spaces subject to continuity of stress and displacement were first studied by Stoneley[1] in 1924 and subsequently by other geophysicists, notably Sezawa and Kanai[2]. In 1947, Scholte[3] discussed the range of existence of such waves and Owen[4] has shown that out of 900 isotropic materials, only thirty combinations provide an interface along which a Stoneley wave can propagate.

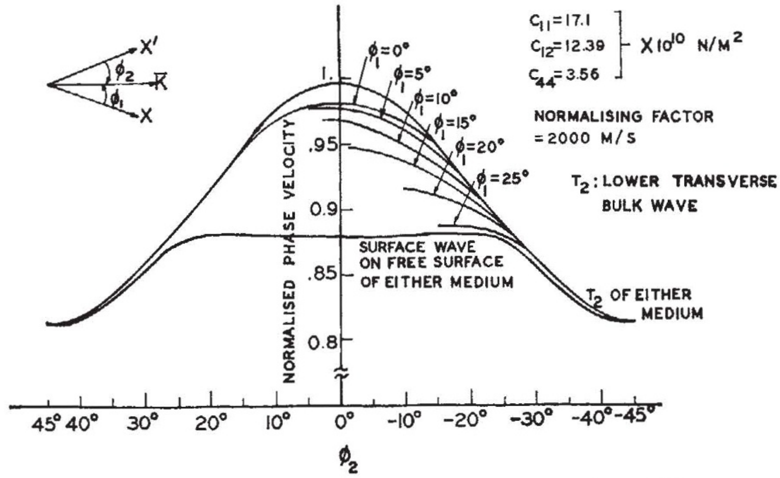

Fig. 1. Velocities $v_s(\varphi_1,\varphi_2)$ of generalized Stoneley waves for $\gamma=1{\cdot}52$.

We report here some calculations for waves at an interface between two anisotropic media, namely, two crystalline half-spaces of cubic symmetry but different orientation. The results indicate that there are many geometrical dispositions which allow a generalized Stoneley wave to be propagated. The interface is chosen as a (0,0,1) plane for each crystal and we consider waves with a real phase velocity or slowness along an arbitrary direction in the interface which makes angles φ_1 and φ_2 with the (1,0,0) axes of the crystals. The angle between the two (1,0,0) axes is thus $\varphi_1-\varphi_2$.

In general, algebraic complexity prohibits an explicit expression for the velocity of the interface wave; a numerical method, however, described by Lim and Farnell[5], has been extended and used to find the appropriate root of the velocity equation. Calculations have been performed for a range of media having stiffnesses c_{11} and c_{12} equal to those of copper, while the remaining shear stiffness c_{44} was arbitrary.

Fig. 1 shows a plot of the values for the velocity $v_s(\varphi_1,\varphi_2)$ of the generalized Stoneley wave for various (φ_1,φ_2) when the half-spaces have stiffnesses such that $2(c_{11}+2c_{12})/3(c_{11}-c_{12})=5{\cdot}92$ and $2c_{44}/(c_{11}-c_{12})=\gamma=1{\cdot}52$. It can be seen that there is a wide range of geometries for which an interface wave may be propagated. Moreover, for all geometries examined, the interface wave velocity is found to lie between the higher free surface (generalized Rayleigh) wave velocity and the slowest bulk wave velocity along the chosen wave normal when referred to the cube axes of either crystal.

It is of particular interest that for any symmetrical case $\varphi_1=-\varphi_2\neq 0$, when the wave vector bisects the angle $\varphi_1-\varphi_2$, the particle displacement in each crystal is elliptic with components attenuated with distance normal to the interface; the resultant displacement at the interface is purely longitudinal, however. For the medium cited above and for $\varphi_1=-\varphi_2=5°$, typical displacement components are shown in Fig. 2 and it is clear that all component amplitudes decay with depth to negligible value in a few wavelengths. As $\varphi_1=\varphi_2\rightarrow 0$, the penetration of the interface wave into the solid increases. Because of the ease of detection of longitudinal displacement, this type of wave may prove useful in devices.

Fig. 1 also shows that each curve of constant φ_1 gradually merges with the slowest bulk wave curve at some $\varphi_2<0$; this is consistent with a result of Burridge[6]. Also, the terminal point of any φ_1 curve corresponds with the merging of the interface wave with the slowest bulk wave; for example, $v_s(20°, -10°)=v_s(10°, -20°)$.

When φ_1 and φ_2 have like signs, the range of $\varphi_1-\varphi_2$ for which interface waves exist appears to be reduced.

Increase of the anisotropy factor γ reduces the angular range $\varphi_1-\varphi_2$ within which interface wave solutions can be found. Such a solution exists, however, for a hypothetical medium in which the shear stiffness c_{44} exceeds the bulk stiffness $(c_{11}+2c_{12})/3$.

The energy fluxes of all the waves have been found to be parallel to the interface.

A more detailed account of this work is in preparation and will be presented elsewhere.

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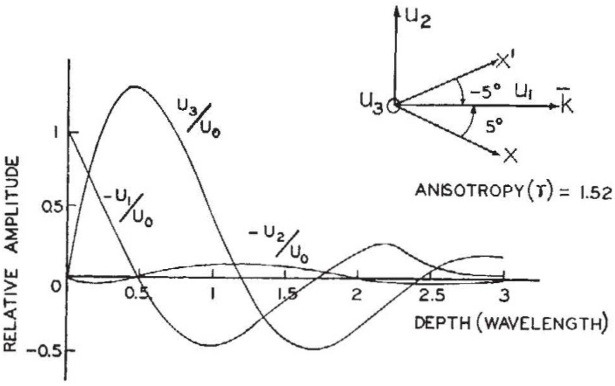

Fig. 2. Amplitude of displacement components U_i as a function of depth. $U_0{}^2=|U_iU_i|_{\text{interface}}$

Influence of Shock Deformation on the Transverse Magnetoresistivity of Polycrystalline Iron

MAGNETORESISTIVITY in a metal has been found to be influenced by anisotropic defect distributions[1,2]. This effect can be obtained only from dislocation distributions which are anisotropic in a region of at most the size of the electronic orbits in the applied magnetic field. In the following work, we investigate the severe deformation region, where the high density of dislocations results in a distortion of crystal symmetry.

The flying plate technique[3] was used in shock loading, allowing for both the magnitude and geometry of the