computed one by one; nonetheless, if examined locally, without being aware of their provenance, they appear 'random'. People have calculated π out to one billion or more digits. One of the reasons for doing this, besides breaking the world record, is the question of whether each digit occurs the same number of times. It looks likely, but remains unproven, that the digits 0 through 9 each occur 10% of the time in a decimal expansion of π. If this turns out to be true, then π would be called a simply normal real number. But although π may be random in so far as it's 'normal', it is far from random in the sense of algorithmic information theory, because its infinity of digits can be compressed into a concise computer program for calculating them.

Now let's consider Omega. Although it has a simple mathematical definition, this definition does not enable us to determine more than a finite number of its digits, which can be shown to be 'normal' in base ten and indeed in any base (for example, in binary, 0 and 1 will occur exactly 50% of the time). Moreover, although the infinite amount of information contained in Omega's digits is algorithmically incompressible, it turns out that Omega is 'computably enumerable', which means that it can be calculated by an infinite process during which one can never know how close one is to the final value. In this way, the halting probability that Omega shares two apparently irreconcilable properties: 'algorithmic randomness' and 'computable enumerability'.

An Omega is computably enumerable because a systematic run of all programs will produce better and better approximations (without being able to compute its digits exactly), and random because it is incompressible; there is no better way to find its digits than by tossing a fair coin. No formal mathematical theory can determine more than a finite number of digits of an Omega. In fact, one can explicitly compute a limit on the number of digits of Omega that a specific theory can determine[4,5]. Berkeley mathematician Robert Solovay[3] has now constructed the 'worst ever' Omega for which no bit can be determined, even with the help of the most powerful formal axiomatic system used by mathematicians, known as Zermelo–Fraenkel set theory.

Each Omega depends on the choice of the computing machine, so there is not just one Omega (as there is only one π), but a class of Omegas. Are there random real numbers other than Omegas that are computably enumerable? This question originates in a 215-page manuscript written by Solovay in 1975 (unpublished manuscript, IBM T. J. Watson Research Center, New York). For years, people working in complexity theory felt that the answer was positive. Solovay imagined a new class of Omega-like numbers that would be distinct from Omegas and would separate them from the other computably enumerable random real numbers (Fig. 1). The intention was to make an Omega-like number share the paradoxical status of an Omega, but not all the properties of a true Omega; Omega-like numbers would then outnumber the Omegas. An Omega-like real number behaves like an oracle: its huge amount of information can be used to compute close approximations for every computably enumerable real number.

In 1998 a first unexpected result was proved: every Omega-like real number is an Omega[7]. The existence of a computably enumerable random real number that is not an Omega became less plausible, but was not ruled out. The last step has been brilliantly accomplished by another Berkeley mathematician, Theodore Slaman[2], who has proved that every computably enumerable random real number is Omega-like, and hence an Omega.

This work reinforces the message of algorithmic information theory that randomness is as fundamental and as pervasive in pure mathematics as it is in theoretical physics. In our opinion it also gives further support to 'experimental mathematics', and to the 'quasi-empirical' view of mathematics which says that although mathematics and physics are different, it is more a matter of degree than black and white[4,5,8]. Physicists are used to working with assumptions that explain a lot of data, but that can be contradicted by subsequent experiments. But mathematicians don't like having to backpedal. Even after Gödel and Turing showed that Hilbert's dream didn't work, in practice most mathematicians carried on as before, in Hilbert's spirit. But now, finally, the computer is changing the way we do things. It is easy to run a mathematical experiment on a computer, but you can't always find a proof to explain the results. So in order to cope, mathematicians are sometimes forced to proceed in a more pragmatic manner, like physicists. The Omega results provide a theoretical underpinning for this revolution. ☐

C. S. Calude is in the Department of Computer Science, Auckland University, Private Bag 92019, Auckland, New Zealand.
e-mail: cristian@cs.auckland.ac.nz
G. J. Chaitin is at the IBM T. J. Watson Research Center, PO Box 704, Yorktown Heights, New York 10598, USA.
e-mail: chaitin@watson.ibm.com

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Daedalus

Fiery compositions

Combustion is seldom a steady process. Most flames oscillate in the sub-audio region; singing flames and some pyrotechnic compositions whistle at much higher frequencies. Daedalus now hopes to tame such oscillations for entertainment purposes. Combustion energies are so high that even a modest rate of burning could generate intense sound.

DREADCO's chemists are now exploring this idea. Their pilot project is simply a firework consisting of alternate layers of fast-burning and slow-burning composition. As the combustion zone moves down through the layers, the gas-pressure generated by burning will be modulated, and the firework will howl out a predictable tone. But how to record an audio signal on such a firework? An electrolytic process seems hopeful. A stick of pyrotechnic composition, dampened to make it conducting, will be passed between a pair of electrodes carrying the analogue audio signal to be recorded. With suitably ingenious chemistry, the density of ionic deposition at each point will control the rate or gas-output of burning at that point. Even with a sonic efficiency of a few per cent (about as good as most loudspeakers), such a 'sound-stick' could deafen its audience with hundreds of watts of sound. The technology would be ideal for public musical performances, and also as a fire alarm. A suitably encoded sound-stick, lit by the blaze, would bellow out its location and appeal urgently for help.

Seeking a milder and cheaper version of the idea, Daedalus recalls the old trick of painting a track on a piece of paper with potassium nitrate solution. When later ignited, the track is revealed by a smouldering glow which travels along it. He proposes to print such a track on paper in directly digital form, as minute dots of oxidant scaled in binary intensity. When lit, the combustion zone would speak the digital signal as it traversed its 'sound track'. Audio burn papers, each with many sound tracks, could be printed in large numbers cheaply and easily.

This elegant technology will combine utter simplicity with a useful sound output. Even a few watts of glow will make a respectable noise. Each track can be played once only, of course, but a paper could carry many tracks, and a music-lover could carry many papers. His inventory of pestilent personal electronic gadgets — mobile phone, pager, calculator and so on — would at least be reduced by one.

David Jones

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