

$2 \times 1$  or  $2 \times 2$  lattice or similar small configurations. Rushbrooke and Scoins do exactly this, while many approximate methods (for example, Kikuchi) attack the combinatorial problem directly, calculating the entropy of the assembly in terms of the small configurations. Woodbury uses an alternative method by writing the problem in matrix form. For a two dimensional  $M \times N$  square lattice a matrix  $\mathbf{P}$  is defined as

$$\mathbf{P}(\nu_i, \nu_{i+1}) = \exp\{-[\frac{1}{2}E(\nu_i) + E(\nu_i, \nu_{i+1}) + \frac{1}{2}E(\nu_{i+1})]/kT\}$$

where  $\nu_i$  is a variable describing the state of the  $i$ th column,  $E(\nu_i)$  is the energy of the  $i$ th column and  $E(\nu_i, \nu_{i+1})$  the interaction between neighbouring columns. If each lattice site can take two states then  $\nu_i$  can take  $2^N$  values and  $\mathbf{P}$  is a  $2^N \times 2^N$  matrix. A standard calculation then shows that the partition function is obtained from the largest eigenvalue of  $\mathbf{P}$  (see T. L. Hill, *Statistical Mechanics*, McGraw-Hill, 1956).

Woodbury's contribution is to use the result that if  $\mathbf{P}$  is applied to an arbitrary vector  $\varphi$   $m$  times then as  $m \rightarrow \infty$   $\mathbf{P}^m \varphi$  approaches the eigenvector corresponding to the largest eigenvalue of  $\mathbf{P}$  (unless  $\varphi$  is orthogonal to it). Choosing  $\varphi$  with components  $\exp\{-E(\nu_i)/kT\}$  as a first approximation he then shows, using this result, that the partition function of an  $M \times N$  square lattice  $Z(M \times N)$ , can be written approximately as

$$\ln Z(M \times N) = \frac{1}{2}MN\{\ln Z(2m+3 \times 2m+3) - 2 \ln Z(2m+3 \times 2m+1) + \ln Z(2m+1 \times 2m+1)\}$$

Thus the partition function can be calculated in terms of the partition function of small lattices. With  $m=0$  the calculation is required for a  $3 \times 3$ ,  $3 \times 1$  and  $1 \times 1$  lattice. Woodbury uses this method with  $m=0$  and 1 and compares his results with Monte Carlo calculations for the problem of a hard sphere lattice gas. The agreement is good except in the critical region where most approximations are inadequate.

This method has the merit that the sequence of approximations  $m=0, 1, 2, \dots$  is clear and the calculation straightforward when the partition functions for the small assemblies have been obtained. The calculation of  $Z(5 \times 5)$  had to be done on a computer and  $Z(7 \times 7)$  or  $Z(9 \times 9)$  would be difficult calculations. Woodbury gives no indication how the method would proceed for a three dimensional lattice but it should be possible easily to extend the method.

## NUCLEAR PHYSICS

### Collisions in ADONE

from a Correspondent

THE first successful collisions of electrons and positrons in the Italian storage ring ADONE were reported in *Nuovo Cimento Letters* on May 21 by Professor Fernando Amman and his colleagues. Wideroe first had the idea of making beams of particles collide in 1947; nobody seems to have noticed his work, and the idea was reinvented by G. K. O'Neill in 1956. The problem is how to obtain enough collisions. For electrons and positrons the density of the beams is increased by the damping caused by synchrotron radiation.

Two storage rings of 500 MeV electrons were started in 1959 at Stanford and in the past three years have been used to demonstrate the validity of quantum electrodynamics at high momentum transfers. When

Bruno Touschek emphasized the importance of colliding beams of electrons and positrons he stimulated a great deal of Italian work. Not only can physicists study quantum electrodynamics with  $e^+e^-$  rings, but they can produce any meson resonance of spin and parity  $J^P=1^-$  and produce, in pairs, any particles with a charge or magnetic moment, free of extraneous strong interaction.

A small storage ring, ADA, was built in 1961 and many tests were made on beam dynamics. But the intensities were too small for experiments in high-energy physics. ADONE was then designed to have adequate energy and intensity, and construction started in 1964.

It is still difficult to make beams interact and so storage rings do not work as soon as construction is finished. The beams have a tendency to become unstable at every opportunity and a whole branch of physics concerned with controlling these instabilities has been created.

There is usually a long period between injection of the first beam, injection of the second beam, storage of adequate currents and the achievement of particle collisions of adequate intensity. It is this last step that has been reported by the group at ADONE, so that experiments in high energy physics can now begin. A luminosity, defined as the ratio of counting rate to cross section, of  $3.5 \times 10^{32} \text{ cm}^{-2} \text{ h}^{-1}$  has been achieved at 1,100 MeV. This is close to design and the design intensity is expected to be achieved soon. This energy and luminosity are the highest yet achieved in electron-positron or electron-electron beam collisions.

Two other  $e^+e^-$  storage rings have been providing exciting results in the past two years. Teams from ACO at Orsay, France, and WEPP2 at Novosibirsk in the Soviet Union, announced the first production of  $\rho^0$  mesons, free of strong interactions, eighteen months ago. Later, similar studies of the  $\omega^0$  and  $\varphi^0$  mesons were published (J. E. Augustin *et al.*, *Phys. Lett.*, **28B7**, 508; 1969).

Now that ADONE is ready there should be many more exciting experiments in the next few years. New storage rings of still higher energy and intensity are planned in the United States, Germany and the Soviet Union. Their designers will be encouraged by the success of ADONE. At the Daresbury Nuclear Physics Laboratory, there are plans for a 20 GeV electron synchrotron ring to be filled by the existing synchrotron. The designers should consider seriously the possibility of storing beams in this ring.

## NUCLEAR PHYSICS

### Towards Cheaper Accelerators

from a Correspondent

A SUGGESTION that may ultimately reduce the cost and improve the performance of high-energy particle accelerators has recently been made by N. C. Christofilos in *Physical Review Letters* (**22**, 830; 1969). Christofilos's proposal is essentially a modification of the collective-ion acceleration technique, itself one of the most promising ideas to emerge in accelerator physics during the past few years.

The collective-ion technique involves accelerating a composite system of positively charged ions trapped in the electrostatic potential well at the centre of a ring