in the power output per unit length of a carbon dioxide laser by a factor of nearly three with further papers on this system. The group also announced an argon ion laser with a remarkably improved life-time and higher output power.

Several industrial groups described progress on a holographic memory, consisting of an array of holograms scanned by a light beam, which should be capable of holding 10^8 bits of information. Papers by Basov *et al.* (P. N. Lebedev Physical Institute, Moscow) gave results obtained with semi-conductor logic elements and various types of spatial filtering techniques were also covered.

One of the more interesting possible fields opened up by the laser is that of optical communications. So far no great progress has been made in this direction, but the situation is gradually changing. Two papers arising out of the work of K. C. Kao (STL, Harlow) were concerned with a cladded glass fibre communications system consisting of fibres perhaps 100 μm thick which could have a band width of several gigahertz. Its realization depends on the development of suitable materials with extremely low extinction coefficients.

IMMIGRATION

Branching Process

from our Mathematical Probability Correspondent

I HAVE mentioned before that the number of individuals in successive generations of a branching process (a mathematical model of a population growth process) will, with probability one, eventually become either zero or infinity. But if immigration takes place into the population in each generation, it is possible that the population may be rescued from extinction at one end of the scale and from explosion at the other. It is then a problem to discover how much may be injected by way of immigration to keep a population alive without causing it to explode.

C. R. Heathcote (*J. Roy. Statist. Soc.*, B, 27; 1965) provided an answer to the problem for the Galton–Watson branching process with the additional immigration component in the case when the expected (average) number of offspring per individual is less than one. E. Seneta (*J. Roy. Statist. Soc.*, B, 30; 1968) has recently extended the results to the case when the expected number of offspring per individual is equal to one. Heathcote argued that in this latter case explosion is bound to occur, while Seneta shows that Heathcote's argument is false. Suppose that ξ is the number of offspring per individual, that $p_k = Pr\{\xi = k\}$, $f(s) = \Sigma p_k s^k$ and that $E(\xi) = \Sigma k p_k = 1$. Suppose also that η is the number of immigrants into the population in one generation, that $q_k = Pr\{\eta = k\}$, $g(s) = \Sigma q_k s^k$, and that $E(\eta) = \Sigma k q_k = \mu$. Seneta's conditions for non-explosion (with probability one) are

$$\mu < \infty; \int_0^1 \frac{1-s}{f(s)-s} ds < \infty$$

Seneta's proof, which is both simple and elegant, is perhaps worth sketching. Define Z_n to be the number of individuals in the nth generation and $F_n(s) = \Sigma Pr\{Z_n = k\} s^k$. Then, if $Z_n = k$, $Z_{n+1} = \Sigma \xi_i + \eta$, so that $F_{n+1}(s) = g(s) F_n(f(s))$. Thus

$$F_n(s) = f_n(s) \prod_{i=0}^{n-2} g(f_i(s))$$

where $f_0(s) = s_1$, $f_1(s) = f(s)$ and $f_{n+1}(s) = f(f_n(s))$. Now $Pr\{Z_n = 0\} = F_n(0)$, and since $f_n(0) \to 1$ as $n \to \infty$, $F_n(0) \to \prod_{i=0}^{\infty} g(f_i(0))$. This limit is strictly positive if and only if $\sum_i \{1 - g(f_i(0))\} < \infty$, and this happens when $\mu < \infty$ and $\sum_i \{1 - f(0)\} < \infty$, and in turn this happens if and only if $\int_0^1 \frac{1-s}{f(s)-s} ds < \infty$, a result due to Breny, Reuter and Seneta. The fact that the population may eventually become empty with positive probability, but may also move from the empty state to any other state by immigration, shows that $\{0, \infty\}$ are not the only possible states for the population to be in, in the long run. In fact the state ∞ is not a possible state (a deep result from the theory of Markov chains), and there exists a stationary distribution $\{\pi_k\}$ where $\pi_k =$ limit as $n \to \infty$ $Pr\{Z_n = k\}$, with $\Sigma \Pi_k = 1$ and $\pi(s) = g(s)\pi(f(s))$, where $\pi(s) = \Sigma \Pi_k s^k$. The equation $\pi = g\pi(f)$ is usually impossible to solve explicitly, but it may be used to show that the expected size of the eventual population $(\Sigma k \pi_k)$ is infinite, and Seneta points out that an approximate solution

$$\Pi^*(s) = \exp\left\{\int_s^1 \frac{1-g(t)}{t-f(t)} dt\right\}$$

holds in the case when $p_1 \approx 1$ and $q_0 \approx 1$.

COMPUTING

Centre for Calculations

from a Correspondent

A NEW European centre will open in Paris at the end of this year with the aim of promoting and developing the application of large capacity high-speed computers to the solution of problems concerned with atomic and molecular structure and properties. It will be called the European Centre for Atomic and Molecular Calculations (CECAM).

This centre has been founded because a number of European chemists and physicists feel that Europe has not achieved in this field a success sufficiently comparable with that reached in America. There have been several centres on the other side of the Atlantic where the skill in developing programs has been enhanced by interactions among a considerable group of co-workers. It is hoped that the centre in Paris will be able to draw together a fairly large group, composed of members with varied experience, which can be more effective than any in Europe so far. The object is not to increase the range of the application of existing programs—most of which now originate outside Europe—but to improve the skill in devising new programs and in developing new applications to atomic and molecular problems. Those who attend the centre will then return to their own countries where they can disseminate the additional experience obtained at the centre. It is hoped, however, that they will return periodically to the centre and will continue to cooperate in its work.

CECAM will be at the University of Orsay, about twenty miles from Paris, in buildings provided by the Centre National de la Recherche Scientifique (CNRS). Time will be available on CDC 3600 (64K memory) and IBM 360–50–75 computers. It is hoped that there will be about twenty scientists at the centre at any time. Each scientist will be there for at least