Energy Flux from the 3° K Radiation

WE wish to make precise statements concerning black body radiation, isotropic in a frame S_0, which moves with velocity v in the observer's frame S. This is of interest in connexion with (a) the problem of measuring the Earth's velocity through the 3° K cosmic background radiation field and (b) the problem of assigning a temperature to a moving black body.

Problem (a) has recently been solved[1,2] for the case $h\nu \ll kT$. Although it has not been used in connexion with problem (a), the general solution was given by Pauli[3], and we have used his results in connexion with problem (b)[4]. For example, a proof is given in ref. 2 that in any direction φ (the angle between and photon velocity) the black body radiation remains black body radiation at an effective angle-dependent "temperature" $T = T_0 m$, where $m \equiv \beta\,[1-(v/c)\cos\varphi]$, $\beta \equiv (1-v^2/c^2)^{-1/2}$, and T_0 is the proper temperature. This can be seen from Pauli's equation (381b) for the specific intensity (or brightness)

$$K_\nu = \frac{2h}{c^2}\,\frac{\nu^3}{\exp(mh\nu/kT_0) - 1}$$

It is well known that K_ν and T_0/m are least if one looks in the direction ($\varphi = \pi$) in which the radiation field, or the radiation source, are receding.

Some new remarks which may be helpful can, however, be made. If the specific intensity in S_0 is $K_{0\nu}$, we shall study the effect of the motion through the ratio $X \equiv K_\nu/K_{0\nu}$. We have for the intensity in a given frequency range and in a given solid angle $K_{0\nu_0} = f(\nu_0)$, say, from which it follows that

$$K_{0\nu} = (\nu/\nu_0)^3 f(\nu_0) = m^{-3} f(m\nu)$$

The required factor is then

$$X = f(m\nu)/m^3 f(\nu)$$

For the energy flux (energy received per unit area per unit frequency range) from an extended but small source, we need to incorporate the solid angle $d\Omega$ subtended by the object at the observer for the viewing direction. The correction factor for this energy flux is

$$Y = \frac{K_\nu\, d\Omega}{K_{0\nu}\, d\Omega_0} = \frac{f(m\nu)}{m\, f(\nu)}$$

It is unity for all frequencies and velocities only if $K_{00} \propto \nu$.

Applying this result to black body radiation, Y is not in general unity

$$Y_{\rm bbr} = \frac{\exp(h\nu/kT_0) - 1}{\exp(mh\nu/kT_0) - 1}\, m^2$$

Some typical results are shown in Fig. 1. We note that a receding source produces an enhanced energy flux and

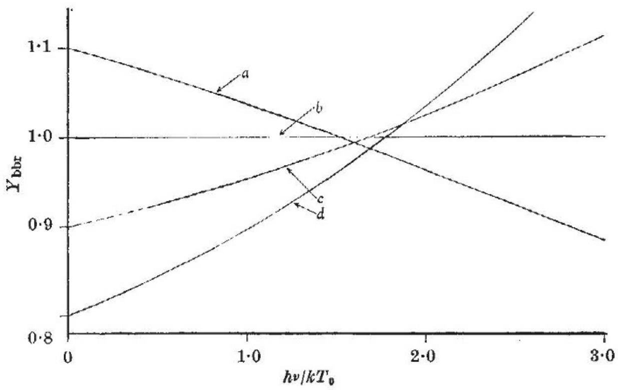

Fig. 1. a, $m = 1{\cdot}1$ (for example, receding source with $\varphi = \pi$); b, $m = 1{\cdot}0$ (stationary source); c, $m = 0{\cdot}9$; and d, $m = 0{\cdot}8$ (for example, approaching source with $\varphi = 0$).

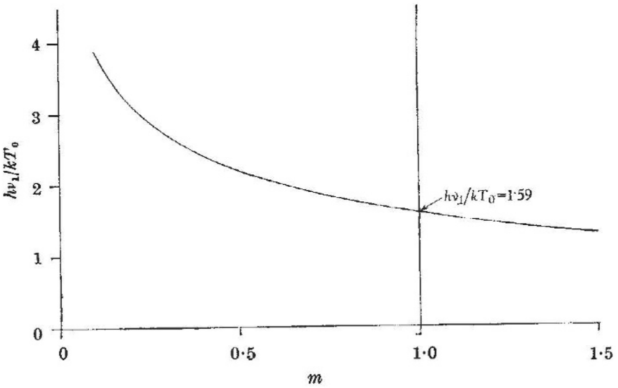

Fig. 2.

an approaching source a lowered flux, for low frequencies. These effects are qualitatively opposite from what would be expected from the Doppler effect alone. The reason is best explained by considering two observers, one at rest in S, the other at rest in S_0, who are at the same point at the moment of observation. The ratio m^2 of the solid angles subtended at the observers is in excess of unity for a receding source by an amount which is independent of frequency. On the other hand, $K_\nu/K_{0\nu}$ approaches $1/m$ from exponentially small values as ν decreases to zero. For receding sources the effect of the solid angle therefore dominates for small enough frequencies. The situation for an approaching source is analogous. The frequency ν_1 below which the "anomalous" behaviour occurs is the solution of

$$(z - 1)\, m^2 = z^m - 1,\ z \equiv \exp(h\nu_1/kT_0)$$

and is shown in Fig. 2.

The anisotropy of the radiation can conveniently be discussed in terms of the ratio

$$A \equiv \frac{K_\nu(\varphi = 0) - K_\nu(\varphi = \pi)}{K_\nu(\varphi = 0) + K_\nu(\varphi = \pi)}$$

For black body radiation ($\alpha \equiv \beta h\nu/kT_0$)

$$A = \frac{\sinh(\alpha v/c)}{\cosh(\alpha v/c) - \exp(-\alpha)}$$

In the case of the 3° K background radiation $v/c \ll 1$ so that the experimentally accessible quantities A and α yield in that case

$$A\,[1 - \exp(-\alpha)]/\alpha = \frac{v}{c}$$

for each frequency. The quantity A is alternative to the "brightness difference", used in ref. 2, and may have some advantages. For given v/c, A increases monotonically with ν from the value v/c.

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