The occurrence of fracture at a definite angle to the transverse plane could be accounted for in terms of the variation of the inherent strength of the material with direction. It is only necessary to assume that the rate of decrease of the inherent strength as this angle increases is greater than the rate of decrease of the normal component of stress across the plane considered.

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## Fracture in Bending, Torsion and Radial Pressure

I HAVE shown[1] that glass rods with normally damaged surfaces, and subjected to sustained loading, fracture in equal times when subjected to numerically equal prior principal stresses, in bending, torsion and radial fluid pressure. The term prior stress refers to the stress distribution which exists before any cracking has occurred. I interpreted these results in terms of quasistatic crack growth with consequent increase in the stress concentration at the ends of the cracks, and I now re-interpret them in terms of work based fracture mechanics theory.

Figs. 1–3 represent a cracked body in three states of loading. $X$ is the external force and $u$ the relative corresponding displacement. In Fig. 2 the prior stress ($\sigma$) is applied so as to open the crack. In Fig. 3 the reversed prior stress is applied so as to keep the crack closed.

Applying the reciprocal theorem to the linearly elastic body we have

Figs. 1 and 2: $$Xu_2 = \int_0^A \sigma \, dA e_1 \qquad (1)$$

Figs. 2 and 3: $$0 = Xu_2 - \int_0^A \sigma \, dA e_2 \qquad (2)$$

Figs. 1 and 3: $$Xu_3 = Xu_1 - \int_0^A \sigma \, dA e_1 \qquad (3)$$

where $dA$ is an element of area of one side of the crack surface and $e$ is the relative displacement in the direction of the prior stress of corresponding points on the crack surface.

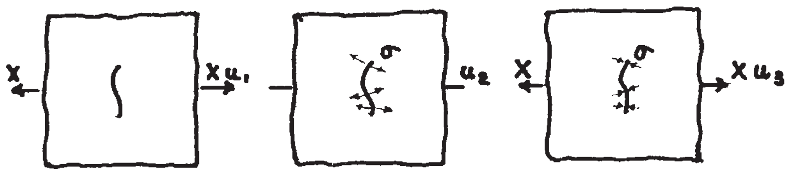


Fig. 1 Fig. 2 Fig. 3

Figs. 1–3. A cracked body in three states of loading.

From equations (1) and (2) we see that $e_1 = e_2$. The strain energy under loading 1 is $1/2\ Xu_1$ and from equation (3) we see that

$$\frac{1}{2} Xu_1 = \frac{1}{2} Xu_3 + \frac{1}{2}\int_0^A \sigma \, dA e_1 \qquad (4)$$

Using equation (2.8) of ref. 2, quasistatic crack growth under loading 1 gives

$$R = \left(\frac{\delta E_1}{\delta A}\right)_X \qquad (5)$$

where $E_1$ is the strain energy in state 1 and $R$ is the fracture toughness.

Now the stress system and hence the strain energy in loading 3 is invariable with the crack area (at constant $X$).

Applying equation (5) we see that for a linearly elastic body

$$R = \frac{\delta}{\delta A}\left(\int_0^A \frac{1}{2} \sigma \, e_1 dA\right)_X \qquad (6)$$

$e_1$ tends to zero at the crack ends, so equation (6) can be rewritten

$$R = \frac{1}{2}\int_0^A \sigma \left(\frac{\delta e_1}{\delta A}\right)_X dA \qquad (7)$$

It should be noted that equations (6) and (7) are valid only when the distribution of external loading, if any, on the crack surface is invariant with crack area. They are not valid for a crack subjected to fluid pressure.

Applying equation (7) to circular glass rods containing cracks so small that the prior stress across the crack faces in bending and torsion is sensibly constant, we see that the prior principal stresses across the crack faces will be equal to an order of accuracy to which $(\delta e_1/\delta A)_X$ is equal for the two loading systems. In the torsion system, equal and opposite prior stresses $\pm\sigma$ exist, the negative stress being applied parallel to the crack. To the order of accuracy to which this negative stress is without influence on the crack opening, we deduce that

$$\sigma\ (\text{torsion}) = \sigma\ (\text{bending})$$

in agreement with the experiments.

A state of radial fluid pressure ($p$) can be regarded as a state of isotropic pressure, on which is superimposed an axial tensile stress, numerically equal to the pressure. Under isotropic fluid pressure, the strain energy is invariant with the crack area, so that the only contribution ($E_1$) to the right-hand side of equation (7) comes from the tensile stress system. Thus

$$R = \frac{\delta}{\delta A}(E_1)_p \qquad (8)$$

where $E_1$ is the strain energy associated with a uniform axial tensile stress numerically equal to $p$.

Thus fracture under radial fluid pressure should occur when the pressure is numerically equal to the uniform axial tensile stress to produce fracture. This deduction is true for cracks of any size. For cracks so small that prior stress variation in bending is negligible over the crack area, we deduce that fracture in bending and tension should occur at the same prior principal stresses, provided statistical effects of crack distribution size are unimportant. Thus the experimental results are consistent with work based on fracture mechanics.

In my earlier communication[1] I attributed delayed fracture of glass to gradual crack growth under stress. Shand[3] has interpreted such crack growth in terms of variation of $R$ with time rate of increase of crack area. For the small cracks with which these experiments are concerned, quasistatic crack growth entails[2]

$$\frac{dR}{dC} = \frac{\dot{c}}{\dot{c}}\frac{dR}{d\dot{c}} = \frac{R}{C} \qquad (9)$$

where $2c$ is a typical crack dimension. When this condition fails unstable cracking ensues.

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