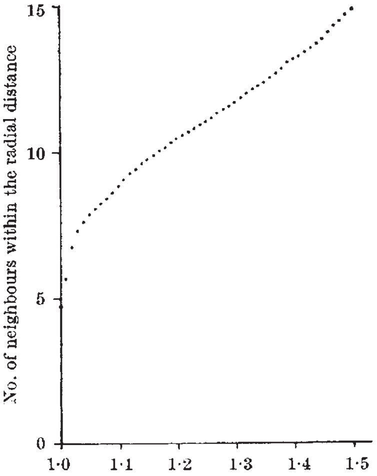

Fig. 4. The cumulative near neighbour distribution function. The average number of neighbours within a radial distance (r) of the central sphere is shown.

region and there are nearly 7 neighbours within 1·02 diameters.

I thank Mr and Mrs A. C. Wright for their helpful discussions. I also thank Professor G. D. Scott for a copy of the original co-ordinates of his packing and Professor J. D. Bernal for the few corrections to these co-ordinates.

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Received January 30, 1968.

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Rational Geometry for measuring Elastic and Viscoelastic Parameters

Recent developments in rubber elasticity theory have led to the recognition that measurements of elastic properties of gels and rubbers near the gel point are of special value[1-3]. The need to develop new methods of measurement has been foreshadowed[2]. We describe here a method under trial for materials in the range 10^3–10^5 dyne cm^{-2} (but which could be extended well beyond this range), the importance of which stems from its rational basis.

Elasticity investigations depend on measuring deformation in response to an applied force. The geometry of the area over which the force is applied to the specimen is important, for it will enter the interpretation of experimental data through the boundary conditions for the differential equations which are relevant. Even in the domain of routine uniaxial tensile modulus measurements, international committees have given much thought to the best geometry of the clamps which transmit the force (see ref. 4).

From both the experimental and theoretical viewpoints, perhaps the simplest testing geometry is that of a rigid sphere embedded in a large reservoir of the material under examination. For materials which adhere to the spherical surface, it is possible to obtain complete theoretical solutions for the relation between the displacement of the sphere ξ and the externally applied force F. Under static conditions the relation for an incompressible material is

$$F = 6\pi\mu a\xi \qquad (1)$$

where μ is the shear modulus and a the radius of the sphere. Thus for materials which exhibit the appropriate tactile properties, measurement of F and ξ leads directly to a value of μ.

Preliminary results for a nickel sphere (radius $a = 0{\cdot}0080$ cm) embedded in a gelatine gel are shown in Fig. 1. The sphere was at distance $> 50a$ from the surface of the gel. An electromagnet capable of generating up to 1,000 gauss at the sphere was used to exert forces of order 1 dyne on the sphere. These forces were calibrated in terms of the "pick-up distance" of the free nickel sphere versus its distance from the tip of the electromagnet. (At the pick-up distance the magnetic force just exceeds the known weight of the sphere.) Displacements of the embedded sphere were measured with a microscope (magnification ×400); they were reproducible for several hours and reversible within experimental error. Fig. 1 shows linear elastic behaviour up to the maximum displacement of about $0{\cdot}7a$. Young's modulus calculated from the slope is $1{\cdot}30 \times 10^4$ dyne/cm^2 (using equation (1) and assuming Poisson's ratio to be 1/2). As a check, a value of $2{\cdot}1 \times 10^4$ was obtained on a sample of the same gel by the surface indentation method[5]. The agreement is thought to be satisfactory because the surface method is much less accurate and liable to give high values of Young's modulus owing to loss of plasticizing action by evaporation of water.

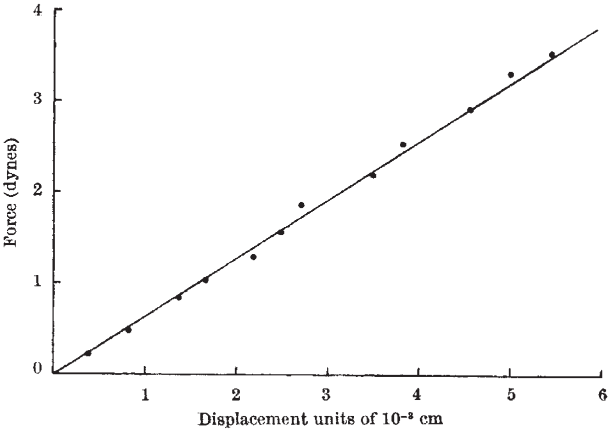

Fig. 1.

One of us[6] has generalized the result (1) to the case of time dependent displacement $\xi(t)$. The principal result is that for an elastic solid, (1) is replaced by

$$F = 6\pi\mu a\xi + 6\pi a^2(\mu\rho)^{1/2}\dot{\xi} - \tfrac{1}{2}M\ddot{\xi} \qquad (2)$$

where ρ is the density and $M = 4\pi a^3\rho/3$ is the mass of material displaced by the sphere. In turn, equation (2) is readily generalized to the viscoelastic case[6], thus permitting analysis of dynamic experiments in terms of viscoelastic parameters.

One of us (J. A. L.) thanks the Plastics Institute education fund for a grant.

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Received January 28, 1968.

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