LETTERS TO THE EDITOR

ASTRONOMY

Arguments for the Presence of Undiscovered Satellites

IN this communication we summarize the results of recent work leading to the prediction of the presence of three new satellites, one each for Jupiter, Saturn and Uranus, and specify those properties of the satellites and their orbits which can be deduced.

In the course of a rediscussion of the characteristics of the satellite systems of the principal planets we showed the following. (*a*) With the exception of the Galilean satellites of Jupiter, commensurability relations of the type

$$\Sigma N_j n_i \simeq 0 \qquad (1)$$

(where n_i is the mean orbital angular rate of the *i*th satellite of a planet, N_j is an integer or zero, and the summation is over all satellites) probably arise by chance and have no physical significance. (*b*) Simple resonance perturbation of particles in Saturn's ring does not explain the observed positions of gaps or relative minima; this conclusion has also been reached by Alfvén[1]. (*c*) The eight inner satellites of Saturn, including the recently discovered Janus[2], obey the following rule

$$n_i + n_{i+2} \simeq (1 + m_i k)\ (n_{i+1} + n_{i+3}) \qquad (2)$$

where n_i is defined as in equation (1), m_i is a small positive integer, k a constant $= 0{\cdot}1505439$, and that such a rule will certainly not arise by chance. (*d*) Similar equations can be formed for the satellite system of Uranus if an extra satellite is postulated in an orbit between the present inner pair, Miranda and Ariel. (*e*) Two solutions pertain to the Jovian system, both of which postulate an extra satellite between Amalthea and Io, with values of $k = 0{\cdot}365514$, $n = 432{\cdot}189$ deg/day or $k = 0{\cdot}262087$, $n = 417{\cdot}802$ deg/day.

We note that, to quite a high degree of exactness, the constant k_q (subscript q denoting the satellite system) is proportional to the mean density, ρ_q, of the corresponding primary. The second of these postulated Jovian satellites fits accurately into this scheme. The relation between k_q and ρ_q (in cgs units) may be expressed as

$$k_q = (0{\cdot}180 \pm 0{\cdot}008)\,\rho_q + (0{\cdot}026 \pm 0{\cdot}022) \qquad (3)$$

To investigate the possibility that an extension of equation (2) might serve to portray the detailed structure of Saturn's ring, we calculated $n_{0,i}$ given by

$$n_{0,i} + n_2 = (n_1 + n_3)\ (1 + 0{\cdot}1505439 m_i) \qquad (4)$$

where n_1, n_2, n_3 are the angular rates of Janus, Mimas and Enceladus, respectively, and m_i a positive integer. It is inferred that these $n_{0,i}$ represent stable orbits, and hence maxima in the particle density in Saturn's ring; and for $m_i \leqslant 10$ this is found to be the case. We find that $n_{0,i}$ must be rejected because its value of 473·288 deg/day is less than that of n_i which is 480·667 deg/day, and that $n_{0,2}$ lies outside the ring with the value 585·23 deg/day. We note, however, that this latter value might represent an undiscovered satellite outside the Roche limit, provided its mean density exceeds 1·3 g cm^{-3}. This hypothesis receives support from Alfvén's proposal that the Cassini division and the relative minimum in the ring at 876·2 deg/day are the cosmogonic shadows of Mimas and Janus, respectively, caused by these satellites sweeping up part of the plasma which later condenses into ring grains, the semi-major axes of whose orbits are 0·65 those of the plasma particles[1]. Alfvén defines the "fall-down ratio" as the ratio between the present semi-major axis of the sweeping satellite's orbit and the radial distance from the centre of the primary to the corresponding ring gap.

The ratios for the pairs (Mimas–Cassini division) and (Janus—876·2 deg/day gap) are 1·541 and 1·492, respectively. We find that the other major gap, the Lyot gap separating rings *B* and *C* and centred on 1,117·8 deg/day, would be the cosmogonic shadow of our proposed satellite, with a fall-down ratio of 1·539 agreeing very well with the ratio for Mimas. The photometric profile of the Lyot gap is at present uncertain, but it seems likely that the proposed satellite has a mass intermediate between those of Mimas and Janus (hence probably a magnitude at mean opposition of about 13), and orbital eccentricity similar to that of Mimas (0·02).

Some elements for the conjectured satellites are summarized in Table 1.

Table 1. PARAMETERS OF THE CONJECTURED SATELLITES OF JUPITER, SATURN AND URANUS

Primary	Conjectured satellite orbital angular rate (deg/day)	Period	Mean distance from primary (equatorial planetary radii)	k_q	ρ_q
Jupiter	417·802	0d 20h 40m 47s	3·66	0·262087	1·334
	432·189	0d 19h 59m 24s	3·58	0·365514	1·334
Saturn	585·23	0d 14h 45m 03s	2·31	0·1505439	0·684
Uranus	214·558	1d 16h 16m 08s	9·03	0·316532	1·60

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[1] Alfvén, H., *Comms. Roy. Inst. Tech., Stockholm*, No. 67–09 (1967).
[2] Dollfus, A., *CR Acad. Sci.*, **264**, 822 (1967).

PLANETARY SCIENCE

Theoretical Model for the Chandler Wobble

EULER is generally credited with having been the first to show that an axially symmetric rigid body, with a fractional difference between the equatorial and polar moments of inertia equal to that of the Earth, could undergo a free nutation with a period of about 300 days. That is, in a body-fixed co-ordinate system, the instantaneous axis of rotation would describe a cone about the polar axis with a 300 day period. It could have been expected that such a motion, even if present primordially, would have been damped almost completely by natural dissipative processes within the Earth. Such a motion would show itself in a periodic variation in astronomic latitude of a given site on the Earth's surface, because the rotation axis moves only slightly with respect to an inertial frame[1]. Despite the expectance of almost complete damping, repeated attempts were made in the nineteenth century to uncover indications of a variation in latitude with a 10 month period. None was definitely established, but in 1891 Chandler[2] announced a variation with a period of 428 days, about 40 per cent larger than predicted. Newcomb soon realized that the period of free nutation for the Earth would be greater than the rigid-body value, because of the fluid nature of the oceans and elastic yielding of the solid earth, and he proposed that Chandler's observations were indeed of the free nutation[3]. Systematic observations of latitude variations have been made since the turn of the century and clearly indicate the presence