BOOK REVIEWS

BIRTH OF THE CALCULUS

The Mathematical Papers of Isaac Newton

Vol. 1, 1664–1666. Edited by D. T. Whiteside. With the Assistance in Publication of M. A. Hoskin. Pp. xlvi + 590. (London and New York: Cambridge University Press, 1967.) 210*s*. net; $40.

When the Royal Society took the decision of limiting the publication of the Newton papers to the correspondence, it was apologetically pointed out that editing of the manuscripts "would require long labour of many patient and profound experts such as it is hard to find today". It is therefore a matter of great satisfaction that one at least of such patient and profound experts has had the pluck to accomplish single handed one of the most important of the tasks shunned by the Royal Society: the critical examination and publication of Newton's mathematical manuscripts. This took him ten years of labour, if we include in the reckoning the painstaking and profound study he carried out, as a preparation, of the whole background of contemporary mathematical knowledge and thinking, in which Newton found the starting point and the inspiration for his own discoveries. The results of this study are embodied in a thesis (*Archive for History of Exact Sciences*, 1, No. 3; 1961) which ranks as a major contribution to the history of seventeenth century mathematics.

With the present volume—the first of eight—the undertaking enters its last stage, and there is every reason to congratulate the author and his collaborator as well as the publishers for a superb achievement, the completion of which will now be impatiently awaited. In format and lay-out the book resembles the volumes of the correspondence, and like the latter is a masterpiece of typography; in details the transcription of the documents is even better than that of the letters. The delicate problems raised by the dating and arrangement of the single items have been solved with great competence and ingenuity. In principle, all the extant material is reproduced, with only trivial exceptions. We thus dispose of every scrap of evidence to help us to retrace the steps that led Newton to his conception of the calculus. In this case, we have even more: in the attempt to present a systematic account of his methods, Newton never managed to proceed further than to drafts, and only one or two of these were eventually published; we are now in a position to read and compare these successive drafts, and to judge, much more easily than from the published pieces, how early Newton acquired a full mastery of the new methods.

The first volume covers only three years, but these were just the decisive ones, 1664–1666. From the notes scribbled in the books Newton used as a student, we can for the first time determine with full certainty the sources of his discoveries, the authors that had the greatest influence on him, and the way in which these influences combined to start him very soon on untrodden paths. This analysis has been done by Dr Whiteside, and he gives us his results in abundant footnotes as well as in the introductions to the four parts (counting the appendix as one part) in which he has distributed the material. Of these, the first two are perhaps the most interesting, although there is much of value also in the last two, devoted to algebra and geometrical optics respectively. From the first part, reproducing Newton's annotations in books, we learn above all of the paramount influence of Descartes' analytical geometry, and especially the commentary added to the Latin edition by F. van Schooten, that able Dutch mathematician who was Huygens' preceptor. It is from the Dutch Cartesian school that Newton got the example of an algorism—a time-saving general prescription—for finding the tangent to a curve from its equation. This rule was limited to the so-called "geometrical" curves, that is, those whose equation could be written in algebraic form.

At the same time, however, Newton learned from the British school, taking its origin in Napier's invention of the logarithms, how to treat non-algebraic functions "mechanically", that is, by series expansions; he had only before him Wallis' crude "interpolation" procedures, and his establishment of the general binomial expansion was a truly creative effort. The crucial point was (to put it in modern terms) to pass from the definite integrals considered by Wallis to the corresponding indefinite integrals: thus was obtained a series in powers of the variable upper limit, with a clear algorism for the computation of the coefficients, instead of Wallis' obscure arithmetical operations. What Newton further learned from Wallis was the method of integration developed by Cavalieri, who, with that other Galilei disciple, Torricelli, had founded a brilliant, though short-lived, school of mathematics in Italy. This method of indivisibles could be readily incorporated in the new calculus as soon as Newton realized that integration and derivation are inverse operations.

The fundamental element in Newton's method, however, the concept of fluxion, came last in the picture. It arose from the need to solve the tangent problem in full generality, that is for the non-algebraic curves called by Descartes "mechanical", because they were mostly defined as loci of the intersection of two straight lines translated in two different directions according to given laws of motion. To construct the tangent in such cases as the diagonal of the velocity parallelogram is not a deep idea: indeed, it is indicated in the pseudo-Aristotelian "Problemata". At any rate, we find a method of construction of tangents on this principle elaborated about 1650 by Torricelli, and further developed, in direct continuation of Torricelli's work, by R. de Sluse and by Roberval. Now, in Newton's notebooks the same method is discussed and applied to various problems, but there is no indication of its source. It is quite possible that he heard about Torricelli's method through Barrow's lectures, though he later declared that he did not remember it. It is also possible that he rediscovered this rather simple approach independently. However this may be, the decisive step he took was to translate the geometro-mechanical construction in analytical terms. How essential this last step was is shown by the case of R. de Sluse who, like Newton, was conversant both with Torricelli's approach and with the algorism for finding the derivative of an algebraic function (which he had found independently): he missed the fluxion idea, however, because it did not occur to him to combine the two methods.

These very fragmentary remarks may perhaps convey some idea of the treasures opened to the historian's view by this monumental publication. Thick as it is, however, this book reflects only a part of the lonely youth's astounding intellectual activity: his studies of Cartesian philosophy and mechanics, his awakening interest in optics, astronomy and chemistry could be documented by volumes of equal importance. Could one hope that Dr Whiteside's brilliant example might stimulate other "patient and profound experts" to undertake for the other domains in which Newton has left his mark a similar labour of love, a labour bringing with it such rich rewards?

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