It is not necessary to print a positive from the recorded hologram, for this only reverses the contrast of the interference fringes. This change of contrast amounts to phase shift of π which does not influence the final intensity in the observed image.

Thus the recorded intensity in the plane F is given by

$$I = |\, a e^{ikr_1 w} + b e^{ikr_2 w} |^2$$

where a and b are Fourier transforms of **A** and **B**, respectively, r_1 and r_2 are the distances of the centres of area of patterns **A** and **B**, respectively, from the optical axis and w is the co-ordinate in the Fourier plane F. Now, when pattern **A** is covered in the input plane and only **B** illuminates the filter at F, the complex amplitude distribution just behind it is given by

$$\begin{aligned} T_F &= b e^{+ikr_2 w} | a e^{+ikr_1 w} + b e^{+ikr_2 w} |^2 = \\ &= b e^{ikr_2 w} [\, |a|^2 + |b|^2] + b \cdot b^* \; a e^{ikr_1 w} + b \cdot a^* \cdot b e^{ik(r_1 + r_2) w} \end{aligned}$$

The lens L_3 takes the Fourier transform of light distribution at F giving the output at O' as

$$O'(z) = [\alpha\beta]_{z=-r_2} + [B^* B^*]_{z=-r_1} \oplus A + [B^* A^* \oplus B]_{z=-(r_1+r_2)}$$

where α is an approximately constant attenuation factor due to $|a|^2 + |b|^2$ and $A_* B^*$ and $A \oplus B$ denote the cross-correlation and convolution of **A** and **B**, respectively. The subscripts $z = -r$, etc., denote the position of the centres of the patterns.

Thus the first term in the last equation is the geometrical image of **B** attenuated by α. The second term occurs at the position where the geometrical image of **A** would have been. This term will also represent the reconstructed image of **A**, provided that $B_* B^*$ tends to a Dirac delta function. Thus in order to recover faithfully the missing pattern from the combination of the two, the illuminating pattern must have a narrow auto-correlation function, ideally tending to a Dirac delta function. The third term is a complicated pattern which may be interpreted as cross-correlation between **A** and **B** convoluted with **B**.

A number of patterns were chosen with various auto-correlation functions ranging from very narrow to very broad. These are shown in Fig. 2 along with their auto-correlation functions. Only some combinations of patterns were used.

Fig. 3 shows the reconstruction obtained from the interference pattern of the first pair, that is, a cross and a triangle. The triangle (Fig. 3*a*) is very well reconstructed when illuminated by the cross, which has a narrow auto-correlation function. On the other hand, the reconstruction of the cross is very poor (Fig. 3*b*). Fig. 4 shows the same for a triangle and a square. Neither of the two objects is well reconstructed, the reconstruction of the triangle being especially poor because the auto-correlation function of the square is very far from a narrow peaked function.

The experimental evidence presented shows that patterns **A** and **B** may be retrieved from the interference of **A** and **B** but with an important qualification. Pattern **A** may only be faithfully recovered if **B**, illuminating the combination, has a narrow auto-correlation function, and vice versa. Thus the method described is not very suitable for character recognition because many letters do not have a narrow auto-correlation function. The method could be used more efficiently with specially designed patterns.

Van Heerden[2] in 1963 proposed a somewhat similar approach for information retrieval. He used what he calls an intensity filter in the Fourier plane; it was simply a positive of a photograph of the Fourier spectrum of a pattern present in the input plane O (Fig. 1). When such a filter is illuminated with a fragment of the same pattern a "ghost" image of the complete pattern may be seen.

Stroke *et al.*[3] have also used a similar filter for what they call "holography with extended sources" and arrived at the same conclusion that the illuminating pattern must have an auto-correlation function tending to Dirac delta function.

I should like to thank Mr D. A. Gregory for the help with experimental work.

B. M. WATRASIEWICZ

British Aircraft Corporation (Operating), Ltd.,
Guided Weapon Division,
Bristol.

Received July 24; revised September 8, 1967.

[1] Gabor, D., *Nature*, **208**, 422 (1965).
[2] Van Heerden, P. J., *Appl. Optics*, **2**, 387 (1963).
[3] Stroke, G. W., Restrick, R., Funkhouser, A., and Brumm, D., *Appl. Phys. Letters*, **7**, 178 (1965); and *Phys. Letters*, **18**, 274 (1965).

Identity for the Product of Two Positive Definite Quadratic Forms

THE problem of generalizing the following basic identity

$$\sum_1^p a^2 \sum_1^p b^2 = (\Sigma ab)^2 + \sum_{i>j} (a_i b_j - a_j b_i)^2$$

given in Hardy, Littlewood and Pólya[1], remains to be considered. We shall show that the generalized Cauchy inequality follows immediately from this generalization. The resulting identity has many fruitful applications in problems of multivariate ratio estimation in sample survey theory.

Let $C = [c_{ij}]$ $(i, j = 1, 2, \ldots, p)$ be a symmetric matrix possessing a non-singular inverse $C^{-1} = [c^{ij}]$ $(i, j = 1, 2, \ldots, p)$. Denote the jth column vectors of C and C^{-1} by c_j and c^j, respectively. Let $H = C^2$, so that H is a $p \times p$ symmetric positive definite matrix and let $z = (z_1, z_2, \ldots, z_p)$ and $u = (u_1, u_2, \ldots, u_p)$ be any two non-null row vectors of length p. Then we have the following identity

$$(zHz')(uH^{-1}u') = (zu')^2 + \sum_{i>j} \{z(c_i uc^j - c_j uc^i)\}^2 \qquad (1)$$

which is the generalization of the basic identity given here, the summation in the last term being over the C_2^p pairs of (i, j).

The proof is quite simple. In matrix form the basic identity can be expressed as

$$aa'.bb' = (ab')^2 + \sum_{i>j} (a_i b_j - a_j b_i)^2 \qquad (2)$$

where a and b are non-null row vectors of length p. Setting $a = zC$ and $b = uC^{-1}$ in the identity (2) and remembering that $H = C^2$ and that the scalars $a_i = zc_i$ and $b_j = uc^j$, the identity (1) will be obtained.

The generalized Cauchy inequality follows by noting that

$$(zHz')(uH^{-1}u') \geq (zu')^2 \qquad (3)$$

equality holding if and only if $zc_i = \theta uc^i$ for all i, where $\theta \neq 0$ is a constant, which implies that we must have $zC = \theta uC^{-1}$ or $zH = \theta u$.

Finally, in identity (1), if we set $M = I$ the identity matrix, and remember that $I = I^2$, we get back to the basic identity.

J. C. KOOP
G. B. GRAY

Dominion Bureau of Statistics,
Ottawa.

Received August 7, 1967.

[1] Hardy, G. H., Littlewood, J. E., and Pólya, G., in *Inequalities* (Cambridge University Press, 1934).