Solar Oblateness

by

I. W. ROXBURGH

Department of Mathematics,
Queen Mary College,
University of London

In this article the author answers criticism, by Dicke and Goldenberg, on his theory of why the Sun is oblate.

RECENTLY, in an article on the oblateness of the Sun¹, I suggested that the observed difference of 35 km between the polar and equatorial radii² could be caused by the effect of uniform rotation on the solar convective zone. Dicke and Goldenberg, in a reply³ refuting this suggestion, did not consider my principal hypothesis, but concentrated only on the weaker peripheral deductions.

My basic suggestion was that the solar rotation interacts strongly with the turbulent convection in such a way that a steeper temperature gradient is necessary in the polar direction than in the equatorial. If the temperature at the base of the convective zone is approximately the same at pole and equator, then the temperature falls to the surface value in a shorter distance in the polar direction than in the equatorial. Thus the surface is oblate.

In support of this hypothesis the following results can be quoted. (1) Investigations of the onset of instability⁴ do show that rotation inhibits convection when rotation and gravity are parallel (the polar case), but not when they are perpendicular. (2) Investigations by Weiss⁵ and myself on turbulent convection in a rotating system give the suggested result, although the magnitude of the effect cannot be accurately predicted because there is no satisfactory theory of turbulent convection. (3) Although rotation can only have a small effect in stable zones, because the ratio of centrifugal force to gravity is small, this need not be the case in turbulent convective zones. In some respects gravity is weakened by a factor of the ratio of the superadiabatic temperature gradient to the actual temperature gradient (which is small compared with unity), so that even a slow rotation could conceivably have a large effect.

Certain deductions can be made from the hypothesis which supports the general idea. In particular, the different temperature gradients in the polar and equatorial directions imply a temperature and thus a pressure difference over a surface of constant density. The resulting latitudinal pressure force has to be balanced by motions other than uniform rotation, and the observed equatorial acceleration of the Sun could be taken as evidence of such an effect. Dicke and Goldenberg consider that their model with a rapidly rotating core provides several mechanisms to explain and maintain the equatorial accelerations while the model I proposed does not. In my original paper I considered a radial angular velocity gradient. This need not necessarily be the case; the actual variation would depend on the latitude dependence of the excess pressure over an equidensity surface which could only be obtained by a more detailed analysis. As far as the maintenance is concerned, Kippenhahn⁶ and co-workers have found that a circulation in the convective zone (in this case driven by the pressure excess) is

capable of maintaining equatorial acceleration in a viscous envelope.

Dicke and Goldenberg list three of my original points for answer. (a) The surface of the Sun is located where the temperature equals the effective temperature; (b) if the solar core were rapidly spinning, the polar flux would exceed by about 1 per cent that at the equator; and (c) the excess oblateness of 27 km is caused by an internal temperature gradient and a large positive radial gradient in angular velocity.

I agree with their statement that the $T = T_{\text{eff}}$ is not the observed solar limb, which is at an optical depth of about 0.004. This does not affect my conclusion because wherever the limb is situated the temperature is negligibly small compared with the temperature at base of the convective zone, and can be taken as zero as far as calculating the oblateness is concerned. Admittedly there is much to be done before we understand the structure of the solar atmosphere, and the exact nature of the adjustment of the atmosphere to the oblate convective zones requires detailed examination.

I agree with Dicke and Goldenberg's doubts over the use of von Zeipel's theorem; however, it is more likely to be valid for the interior than for the surface layers.

I do not state that the oblateness is caused by a radial angular velocity gradient. The oblateness is supposed to be caused by the interaction of rotation and convection. This in turn requires some motion other than rotation. Whether this excess motion is differential rotation in a radial sense or an angular sense, or primarily a circulation, has yet to be resolved.

Finally, one should point out that Goldreich and Schubert (work to be published) have recently found that the Rayleigh-Taylor instability occurs in a stably stratified star, and that therefore the angular velocity must be independent of distance from the equatorial plane and that the angular momentum per unit mass must increase away from the rotation axis. This prevents the development of radial variation in angular velocity and ties the rotation of the solar core to that of the envelope. It is therefore difficult to see how the inside of the Sun could be rotating faster than the outside as required in the Dicke model.

Received November 1, 1967.

¹ Roxburgh, I. W., *Nature*, **213**, 1077 (1967).

² Dicke, R. H., and Goldenberg, H. M., *Phys. Rev. Lett.*, **18**, 313 (1967).

³ Dicke, R. H., and Goldenberg, H. M., *Nature*, **214**, 1294 (1967).

⁴ Chandrasekhar, S., in *Hydrodynamic and Hydromagnetic Stability* (Oxford University Press, 1961).

⁵ Weiss, N. O., *Phil. Trans. Roy. Soc.*, **256**, 99 (1964).

⁶ Kippenhahn, R., in *Atti del Convegno Sui Campi Magnetici Solari* (edit. by Barbera, G.) (Firenze, 1966).