LETTERS TO THE EDITOR

PLANETARY SCIENCE

Shape of the Moon from the *Orbiter* Determination of its Gravitational Field

An analysis of the perturbations of the American *Orbiter* 1, revolving since August 14 in a 200 min orbit close to the Moon, has led to a satisfactory determination of the principal characteristics of the lunar gravitational field[1], and it is of interest to compare these with previous predictions based on plausible physical assumptions. In a previous investigation[2] one of us predicted the coefficients of a harmonic expansion of the lunar gravitational potential on the assumption that the mass of the Moon is distributed homogeneously. The volume of the Moon is defined by a surface obtained from the hypsometric data of the Aeronautical Chart and Information Centre of the U.S. Air Force[3]. We believe these data to be the best available at the present time, because the use of high-resolution sequential photographs from Pic du Midi enabled the ACIC investigators to minimize the atmospheric effects, which are the principal source of errors.

Direct determinations of the coefficients of the lunar gravitational field are now available, so that it is possible to invert the problem, and determine the shape of the Moon on the assumption that it is a homogeneous body. (Small deviations from homogeneity, as shown for example by the form of the lunar profiles deduced from the eclipse data[4], do not affect the results in any significant manner.) We find that the principal features of the global shape of the Moon, as deduced from the *Orbiter* data, should be described by the following tesseral harmonics:

$$\begin{aligned} Y_2(\lambda,\beta) = {} & 0{\cdot}30 - 0{\cdot}89 \sin^2\beta - (0{\cdot}06 \cos \lambda - 0{\cdot}11 \sin \lambda)\sin \beta \cos \beta \\ & + (0{\cdot}21 \cos 2\lambda + 0{\cdot}01 \sin 2\lambda)\cos^2\beta, \end{aligned} \tag{1}$$

$$\begin{aligned} Y_3(\lambda,\beta) = {} & -0{\cdot}60 \sin \beta + 1{\cdot}00 \sin^3\beta + (0{\cdot}02 \cos \lambda + 0{\cdot}11 \sin \lambda)\ (5 \sin^2\beta - 1) \cos \beta \\ & + (0{\cdot}19 \cos 2\lambda + 0{\cdot}56 \sin 2\lambda)\sin \beta \cos^2\beta \\ & + (0{\cdot}07 \cos 3\lambda - 0{\cdot}04 \sin 3\lambda)\cos^3\beta, \end{aligned} \tag{2}$$

$$\begin{aligned} Y_4(\lambda,\beta) = {} & 0{\cdot}12 - 1{\cdot}22 \sin^2\beta + 1{\cdot}42 \sin^4\beta \\ & - (0{\cdot}04 \cos \lambda - 0{\cdot}16 \sin \lambda)\ (7 \sin^2\beta - 3)\sin \beta \cos \beta \\ & + (0{\cdot}06 \cos 2\lambda + 0{\cdot}32 \sin 2\lambda)\ (7 \sin^2\beta - 1) \cos^2\beta \\ & + (0{\cdot}07 \cos 3\lambda + 0{\cdot}04 \sin 3\lambda)\sin \beta \cos^3\beta \\ & + (0{\cdot}04 \cos 4\lambda - 0{\cdot}14 \sin 4\lambda)\cos^4\beta \end{aligned} \tag{3}$$

Numerical values are in kilometres, as functions of the selenographic latitude β and longitude λ, as customarily defined.

Comparison of these expressions with the results of a previous harmonic analysis of the ACIC data[5] shows close correspondence. Strictly speaking, the shape of the surface does not uniquely define the gravitational field emanating from its interior (for the latter is a volume, rather than a surface, property). In order to translate one into the other an explicit assumption on internal mass distribution must be made. A self-gravitating body of a mass as small as that of the Moon, however, cannot depart much from homogeneity, for its self-compression is still marginal, and it is unlikely that the close agreement between our present equations (1)–(3) and their equivalents in Table 4 of reference 5 signifies anything other than the homogeneity of the Moon. It should, therefore, be possible to determine the essential features of the mean selenoid more accurately from its gravitational field than from hypsometric measurements, as the latter are essential only for determinations of such deformations as would be described by harmonics of very high orders. For the Earth, the internal structure of which is more complicated, a close correspondence between volume and surface properties does not seem to occur, but the recent *Orbiter* results disclose that for the Moon this is probably the case.

The results available so far[1] also reveal several other properties of the lunar globe which we wish to mention.

First, the fact that the observed coefficients of the lunar zonal harmonics Y_j for $j = 2$, 3 and 4 are not of the same algebraic sign (their values for $j = 2$ being positive, and negative for $j = 3$ and 4) disposes once for all of the possibility that the underlying deformations may constitute "frozen tides" caused by the attraction of the Earth or any other body—now or at any time in the past; if this were true, the coefficients of all zonal harmonics would have the same sign. In addition, the successive absolute values of the coefficients of the second, third and fourth harmonic diminish only by a factor of about two which implies on the tidal hypothesis that the disturbing body is located at a distance of not more than two lunar radii from its centre of mass; and this rules out the Earth (the radius of which is four times that of the Moon) altogether.

Second, equations (1)–(3) show that the maximum difference in elevation represented by the second, third and fourth harmonic distortion does not exceed 2·2 km. Local level differences exceeding this limit are known to exist on the surface of the Moon, but even disregarding them we conclude that the lunar globe must possess mechanical strength sufficient to sustain load differences amounting to at least 2·2 km of surface material over large areas.

C. L. Goudas
Z. Kopal*
Z. Kopal†

Boeing Scientific Research Laboratories,
Seattle, Washington.

* Normally at the University of Manchester.
† Normally at Stanford University.

[1] Michael, W. H., Tolson, R. H., and Gapcynski, J. P., *Science*, **153**, 1102 (1966).
[2] Goudas, C. L., in *Advances in Astronomy and Astrophysics*, edit. by Kopal, Z., 27 (Academic Press, New York, and London, 1966).
[3] Meyer, D. L., and Ruffin, B. W., *Icarus*, **4**, 513 (1965).
[4] Carson, D., Davidson, M., Goudas, C. L., Kopal, Z., and Stoddard, L. G., *Icarus*, **5**, 334 (1966).
[5] Goudas, C. L., *Icarus*, **4**, 528 (1965).

Odd Zonal Harmonics in the Earth's Gravitational Potential, determined from Fourteen Well Distributed Satellite Orbits

The zonal harmonics in the Earth's gravitational field indicate its variations with latitude, averaged over all longitudes. The odd zonal harmonics, of degree 3, 5, 7, ..., specify that part of the gravitational field which is not symmetrical about the equator and gives rise to the "pear-shape" effect.

The coefficients J_n of the zonal harmonics are defined by expressing the Earth's gravitational potential U at an exterior point, distant r from the Earth's centre and having geocentric latitude φ, in the form

$$U = \frac{GM}{r}\left\{1 - \sum_{=2}^{\infty} J_n \left(\frac{R}{r}\right)^n P_n(\sin\varphi)\right\},$$

where G is the gravitational constant, M the mass of the Earth, and R the Earth's equatorial radius. $P_n(\sin\varphi)$ is the Legendre polynomial of degree n and argument $\sin\varphi$. GM is taken as 398602 km³/sec² and R as 6378·163 km.