are devoted to a lucid and rigorous review of the elements of the calculus of variations. The remainder of the book is a collection of twenty-five important papers published in recent years; about half these papers are by Prof. Miele and his associates, the rest are by well-known authors including R. T. Jones, C. Ferrari, G. V. R. Rao, K. G. Guderley, W. D. Hayes, A. J. Eggers and G. G. Chernyi. All the papers, each designated as a separate chapter, have been extensively edited to ensure uniformity of formulation, standardization of notation and consistency of style—a creditable achievement.

Unfortunately groups of these papers are so closely related in content that for each paper to be treated separately leads to a mathematical sameness and an extremely slow development of the main points of interest. For example, six chapters, or papers, are concerned with the determination of the optimum shapes of bodies of revolution in Newtonian hypersonic flow; different initial assumptions are made in each case: slenderness is first assumed without skin friction, then with constant skin friction coefficient, then with variable skin friction coefficient and then the exercise is essentially repeated for non-slender bodies. There is insufficient mathematical difference between these various aspects to warrant separate chapters. From a practical point of view each chapter in this sub-group is made obsolete by the slightly more generality of the following chapter. The ultimate disillusionment comes in the next section in the book where it is established that bodies with non-circular cross-sections can have lower drags than optimum bodies of revolution. Some discussion of the philosophy of the formulation of optimal problems would have been welcome.

This final point leads on to a further comment. Only those problems are discussed which can be formulated within the framework of the calculus of variations. Thus the brilliant concept of the Busemann supersonic biplane is not mentioned. Neither is there any reference to the vast volume of literature on the transonic and supersonic 'area' rule techniques for minimizing drag of wing-body combinations. It might be argued by Prof. Miele that his terms of reference are not intended to cover such topics, but nevertheless the title of the book does suggest considerable generality.

In conclusion, recognizing the book as a text on the calculus of variations with some applications to the determination of optimum aerodynamic shapes, it is a valuable contribution to aeronautical literature. But it is not so comprehensive as the title implies. In my opinion it could have been condensed with, one hopes, a corresponding reduction in price. G. J. HANCOCK

OPERATORS AND TRANSFORMS

The Dynamics of Linear and Non-Linear Systems

By P. Naslin. Translated from the French. Pp. xxvii + 586. (London and Glasgow: Blackie and Son, Ltd., 1965.) 105*s*. net.

It is now many years since Gustav Doetch showed that the Heaviside operational calculus could be given formal justification in terms of the Laplace transform, which is now accepted as the appropriate tool for solving problems concerning linear systems set in motion at $t = 0$, the subsequent history of which is to be determined. As such it now features in the curriculum of almost all engineering courses of degree or comparable standard.

It therefore seems a pity that the author of *The Dynamics of Linear and Non-Linear Systems* relegates it to a brief appendix. At one place he objects that it involves initial conditions, at another that it is too powerful a tool for the job to be done.

When an input and an output function are connected by a linear differential equation with constant coefficients he replaces d/dt by p and expresses the ratio output to input as a rational function of p which he calls the transmittance. It includes both impedances and admittances when the quantities are electrical.

In a note on the symbols s and p he says that s has the specific meaning of the complex variable of Laplace transform theory, whereas p simply stands for the operator d/dt and thus has a purely symbolic value. The author is right that the transmittance is not a Laplace transform, but it seems just as much a misuse of terms to call it, as he does, a 'p-transform'. Equally to regard p as meaning simply d/dt is absurd when the poles and zeros of a transmittance are considered.

The only logical point of view is that the transmittance is the transmission ratio when the input and output depend on time through the exponential factor $\exp pt$. This justifies at once replacing d/dt by p and the differential equation may be recovered by reversing the steps. The steady-state transmission ratio is obtained by setting $p = j\omega$, and, replacing p by s or not, the transmittance function coincides with the Laplace transform of the output when the input is a unit impulse.

Linear systems are represented by block diagrams. Thus if a battery e.m.f., E, is the input to a simple potential divider with resistances R_1 and R_2 in series, the output being the potential v across R_2, this is represented by applying E through an adder to a multiplier $1/R_1$ which produces the current i. This applied to a second multiplier R_2 gives v, while a feed-back path with multiplier -1 runs from v back to the adder so that $E-v$ is actually applied to the first multiplier.

Instead of multipliers there may be integrators, differentiators or more complicated systems. The block diagram may be modified in a variety of ways without altering its overall transmittance. It may be arranged so that only adders, multipliers and integrators are used when it gives the lay-out for an analogue computer simulation of the system. Its use is illustrated in a variety of examples always with application to control systems in mind. The block diagram is easily converted to a signal flow graph for which Mason's formula expresses the transmittance in terms of the transmittances of open paths and closed loops on the graph.

Transients and the sinusoidal steady-state are discussed very thoroughly in close relation to each other and to stability and damping. There is a short chapter on sampled data systems.

The later chapters treat non-linear systems but the emphasis is still on situations where linear theory can be applied: first, the so-called describing function procedure when the effects of harmonics above the first may be neglected, and secondly, phase plane methods applied mainly to problems in which different linear approximations may be used in different ranges of the variables.

In this part of the book some of the figures are not so accurate as they purport to be, possibly through being copied by a draughtsman unfamiliar with the problems represented. The layout of the text and figures is attractive, but for a rather large number of trivial printing errors such as a symbol missing from a formula.

In one respect there is a curiously old-fashioned air about this book. Laborious step-by-step graphical and tabular solutions are described, sometimes with the comment that all the calculations were done on a slide rule. Surely, in these days, every design office concerned with this type of problem has a digital computer available, and some work on numerical analysis features in all engineering courses so that it should scarcely be necessary to devote a final chapter to describing the simplest Euler and modified Euler methods of solving differential equations.

Despite its faults, this book contains a lot of useful and interesting material particularly relevant to the design of control systems. A. W. GILLIES