content of the air, and is difficult to estimate with much confidence, the total depletion of the direct solar beam may amount to as much as 10 per cent.

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PHYSICS

Sub-resonant Response of a Mechanical System parametrically excited at its Resonant Frequency

IN the course of an attempt to improve the response of an inertial seismometer to very-low-frequency frame movements, it appeared worthwhile to investigate the response of a spring-mass system subjected to parametric excitation[1]. The parametric excitation or 'pumping' consisted of modulating the restoring force on the mass by means of an electromechanical spring[2] driven from a sinusoidal current source. In the initial work the restoring force was varied at twice the resonant frequency of the system as is usually done in a degenerate parametric device. It was found both from theory[3] and experiment that this will increase the relative motion between mass and frame only for inputs with a frequency at or near the resonant frequency, and that the input must bear the proper phase relationship to the pump. There is no improvement for frequencies much below resonance.

Pumping at twice the resonant frequency is associated with the critical value, $a = 1$, in Mathieu's equation, $\ddot{x}(\alpha) + (a - 2q\cos 2\alpha)x(\alpha) = 0$, which governs the stability of such systems[4]. One would also expect to obtain gain near the other critical values, $a = 4, 9, 16$, etc., which correspond to pumping at the resonance frequency, two-thirds of the resonance frequency, half the resonant frequency, and so forth.

These phenomena were investigated experimentally and with a computer. Both sets of data confirmed that pumping at the resonant frequency of the system greatly increased the relative motion between mass and frame. In particular, it was found that this increase in motion can be made very large for input frequencies much lower than the resonant frequency of the device; and the increase remains constant and independent of phase relationships down to zero frequency. It is believed that this phenomenon has not been noticed before.

Table 1

	Pendulum A $T_0 = 0{\cdot}57$ sec	Pendulum B $T_0 = 7{\cdot}5$ sec
Period of input	57 sec = 100 T_0	6·25 min = 50 T_0
Motion of mass without parametric excitation	1	1
Motion of mass with parametric excitation	875	1,250

The experimental results are given in Table 1. They were obtained with two different horizontal pendulums (seismometers) with free periods, T_0, of 0·57 and 7·5 sec. These were modified to permit modulation of their period[3] with the spring of time-variable stiffness. Both were pumped at their free period and driven by inputs with periods many times greater than their free periods. In each case it was found that parametric excitation at the resonant frequency could produce as much as three orders of magnitude increase in the motion of the mass with respect to the frame.

An observatory-quality 'parametric seismometer' utilizing this principle is being constructed for the purpose of recording very-long-period vibrations of the Earth.

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Effect of Magnetic Dilution on the Paramagnetic Behaviour of an Ni^{++} Ion in the Crystal of $Ni(SO_4)_2(NH_4)_2 6H_2O$

IT is well known that the symmetry and intensity of the electric field arising out of an axially distorted octahedral water cluster surrounding a paramagnetic ion (M^{++}) determines its magnetic anisotropy. When paramagnetic Tutton salts are magnetically diluted by partially replacing M^{++} ions by Zn^{++} or Mg^{++} ions, the charge distribution is likely to remain the same for different concentrations of M^{++} ions in the mixed crystal. Consequently, the magnetic anisotropy per gram is expected to be almost independent of concentration in the mixed crystal.

Joglekar[1] did not detect any effect of magnetic dilution on the magnetic anisotropy of Tutton salts. Paramagnetic resonance investigations by Griffiths and Owen[2] on the magnetically dilute salt (Ni.Zn) $(NH_4SO_4)_2 6H_2O$ showed that up to a dilution of Ni : Zn = 1 : 50, the crystal field axes and crystal field splitting at 90° K are very similar to those of the concentrated salt at 90° K. Bose *et al.*[3], however, found that the crystal field about Ni^{++} ion in nickel Tutton salts is very sensitive to structure and temperature.

In order to test the foregoing points, we have measured the magnetic anisotropy of mixed crystal, of the type $A + nB$ (where A is $Ni(NH_4)_2(SO_4)_2 6H_2O$, B is $Zn(SO_4)_2(NH_4)_2 6H_2O$ or $Mg(SO_4)_2(NH_4)_2 6H_2O$, and n is the number of molecules of B associated with one molecule of A in the mixed crystal) by the well-known method of Krishnan and Banerji[4]. The temperature variation of the anisotropy of the crystal ($A + 15{\cdot}3\ B_1$) was measured by following the method of Krishnan *et al.*[5]. The temperature was controlled by a cryostatic device due to Bose *et al.*[6].

The crystal anisotropy so obtained is converted into ionic anisotropy ($K_\perp - K_\parallel$), following Mookherji[7]. These are shown in Table 1. The temperature variation of ($K_\perp - K_\parallel$) is shown in Table 2. Here, $K_\parallel$ represents the ionic susceptibility along the axis of symmetry of the cluster and $K_\perp$ that normal to it, expressed in C.G.S. electromagnetic units.

It is clear from Table 1 that with increase of dilution, that is, decrease of concentration of Ni^{++} ion, the anisotropy of the cluster ($K_\perp - K_\parallel$) increases. This clearly demonstrates that the magnetic anisotropy is structure sensitive.

Following Mookherji[7], we have calculated $-(\alpha_\perp - \alpha_\parallel)$, the difference between the crystal field coefficients along the symmetry axis of the cluster and that normal to it from ($K_\perp - K_\parallel$) values given in Table 2. It is observed

Table 1

Crystal	A	$A + 3{\cdot}886B_2$	$A + 15{\cdot}14B_1$	$A + 15{\cdot}3B_1$	$A + 25{\cdot}22B_1$
% of Ni^{++} ion	100·00	3·24	0·91	0·89	0·56
$(K_\perp - K_\parallel)10^6$	186	229	342	349	355

Table 2

Temp ° K	310	230	150	110	80
$(K_\perp - K_\parallel)\ 10^6$	349	448	714	987	1,363
$-(\alpha_\perp - \alpha_\parallel)\ 10^6$	25·7	23·5	21·8	19·5	19·9