

In this special and rather unlikely instance an analysis of  $Q(V)$  will yield the function  $N(E)$ . In experiments on secondary electron emission, the source of secondary electrons is usually situated at the centre of a collecting sphere, and all electrons are emitted radially and experience the same retarding potential. The energy distribution measured in such experiments is the quantity  $\int_0^{2\pi} N(E, \sim) d\Omega$ . Such an energy distribution cannot be

found by analysing the characteristic of the spherical vacuum chamber, and the advantages of such a chamber over a parallel plate arrangement are illusory.

The analysis given here is a corollary of my investigations<sup>5</sup> into the behaviour of a parallel plate vacuum chamber. I thank Dr. J. R. Greening for his advice, and the Faculty of Medicine of the University of Edinburgh for supporting the investigations.

K. J. RANDLE

University of Edinburgh,  
Department of Medical Physics,  
The Royal Infirmary,  
Edinburgh, 3.

<sup>1</sup> Greening, J. R., *Brit. J. Radiol.*, **27**, 163 (1954).

<sup>2</sup> Taylor, L. S., *Brit. J. Radiol.*, **24**, 67 (1951).

<sup>3</sup> Burlin, T. E., and Husain, S. R., *Nature*, **204**, 563; **204**, 1078 (1964).

<sup>4</sup> Hachenberg, O., and Brauer, W., *Advances in Electronics*, **11** (1959).

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### Response of Time-dependent Materials to Oscillatory Motion

A CONSIDERABLE amount of attention has been directed in the past to the rheological behaviour of suspensions of deformable particles when subjected to oscillatory motion, with the result that there is now a reasonable body of theory and experimental data relating to the gross behaviour. However, there appears to be little information available for suspensions of undeformable particles such as the clay-water system reported here, although they may exhibit very interesting behaviour.

As an illustration we consider an equation of state of the form:

$$P'_{ij} = 2\mu(t)e_{ij} \quad (i, j = 1, 2, 3) \quad (1)$$

where the total stress is given by:

$$P_{ij} = P'_{ij} + g_{ij}P \quad (2)$$

and  $P$  is an arbitrary isotropic stress,  $e_{ij}$  is the strain rate tensor.

In laminar shearing the time-dependent viscosity may take the form:

$$\mu(t) = \mu_0 + \int_{-\infty}^t \frac{df}{dt'} (I[t']) M(t-t') dt' \quad (3)$$

$\mu_0$  is the initial viscosity,  $f(I)$  is a suitable function of the quadratic strain rate invariant, for example:

$$f(I) = 1 - \exp\left(-\frac{1}{2} \sigma e_{ij} e_{ij}\right) \quad (4)$$

and  $M(t-t')$  is a memory function defined by,

$$M(t-t') = \int_0^{\infty} \frac{R(\Pi)}{\Pi} \exp\left(-\frac{[t-t']}{\Pi} \Pi d\Pi\right) \quad (5)$$

In equations (3) and (5)  $R(\Pi) d\Pi$  is the contribution to the viscosity deficit at the current strain rate  $e_{ij}(t)$  after an indefinitely long period at that strain rate by all the elements of flow with relaxation times between  $\Pi$  and  $\Pi + d\Pi$ .

It may be shown that if the memory of the material is short so that higher moments of the relaxation spectrum ( $Rn$ ) than the first are negligible where:

$$Rn = \int_0^{\infty} \Pi^n R(\Pi) d\Pi \quad (6)$$

then equation (3) takes on the simpler form:

$$\mu(t) = \mu_0 - R_0 f(I) + R_1 \frac{df}{dt} (I) \quad (7)$$

where:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u^i \frac{\partial}{\partial x^i} \quad (8)$$

It is noted that the relaxation spectrum need not be either wholly positive or negative.

Restricting attention now to oscillatory shearing motion in which the shear rate  $\dot{\gamma} = \dot{\gamma}_0 \cos \omega t$  and retaining only the first term in  $\dot{\gamma}$  in (4) then the shear stress  $\tau$  is given by:

$$\tau = \left[ \mu_0 - \frac{R_0 \sigma}{2} \dot{\gamma}_0^2 - R \sin(2\omega t + \varphi) \right] \dot{\gamma}_0 \cos \omega t \quad (9)$$

It is clear from (9) that the shear stress may contain odd harmonics of the shear rate frequency. The viscous shear stress may be zero either when the shear rate is zero or when the viscosity is zero, that is:

$$\mu_0 - R_0 f(I) + R_1 \frac{df}{dt} (I) - \dots = 0 \quad (10)$$

An exactly similar argument applied to a time-dependent yield criterion shows that the shear stress may contain even harmonics from this source and the yield stress is zero under an analogous criterion to equation (10).

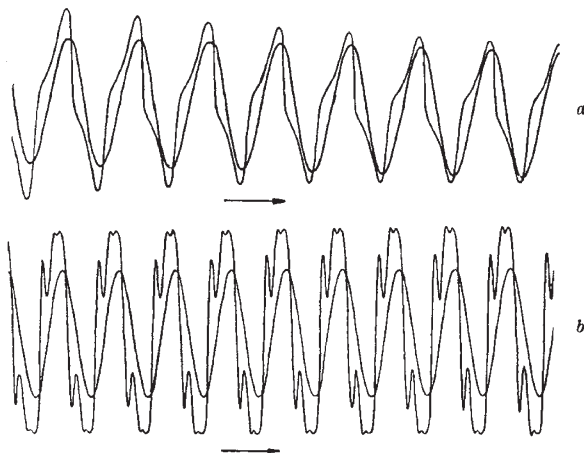


Fig. 1. Errol clay in distilled water (30 g/20 ml.). Shear amplitude, 0.141. (a) 0.0792 c/s, stress amplitude diminishes with time, single peak; (b) 7.92 c/s amplitudes of stress increase with time, multiple peaked. Apparatus, Weissenberg rheogoniometer; natural frequency of torsion head, 42.4 c/s

Examples of the response of a clay-water system are given in Figs. 1 a and b. Horizontal displacement of the shear stress relative to the strain rate may be due to elasticity, or time-dependency of the viscosity or yield stress. It is interesting to note that at the low frequency the stress amplitude diminishes with time whereas at the high frequency the stress amplitudes increase with time, suggesting that the associated relaxation spectra may not be entirely positive.

A more comprehensive account of these phenomena will be published elsewhere.

J. HARRIS

Department of Mechanical Engineering,  
Queen's College,  
Dundee.