LETTERS TO THE EDITOR

ASTROPHYSICS

Anomalous Velocities in Quasar 3C273

ESTIMATES of the size of quasi-stellar object 3C273 ranging from $\sim 1,000$ light years to ~ 1 (ref. 1) light year when coupled with the first observations of Smith and Hoffleit² regarding the apparently uniform and simultaneous pulsations giving rise to luminosity fluctuations with period of ~ 10 years and with recent reports of a ~ 1 (ref. 3) month fluctuation period present the possibility of the existence of anomalous velocities between the light velocity c and $\sim 100c$. Indeed it would appear that the discovery of smaller fluctuation periods would correspondingly lead to velocities of an increasingly anomalous nature despite consistency of any given size estimate with usual velocity range and a given fluctuation period.

It may consequently be of interest to consider the possibility that the anomalies may be attributed to a hyperdimensional effect were it actually to exist as reported not very long ago by generalizing the well-known Einstein, Infeld and Hoffman (E.I.H.) method to the problem of motion in five space⁴. The principal results may be summarized by stating the following partial differential equations of motion in the Newtonian approximation for 'particles' characterized by parameters M_j , Q_j proportional to the mass and charge respectively of the j th 'particle' while P_j is another parameter characterizing the j th 'particle' all measured in length units:

$$f_j^{AB} \xi_{j,AB} = -\sum_i \ddot{r}_{ij} [f_i^{AB} f_{j,AB} + (4/9)(\eta_{AB} f_i^{AB})(\eta_{AB} f_j^{AB})] \\ r_i^3 r_{ij} f_{j,B}^{AB} = 0$$

$(\)_{,A}$ denotes partial differentiation with respect to x^A . ξ_j is the ordinary three-space vector for the j th particle while $r_{ij} = \xi_i - \xi_j$. A and B take on only the values of 0 or 4. Such scripts are raised or lowered via the Galilean metric tensor $\eta_{AB} = \eta^{AB}$ in the usual way. The f 's are defined to be:

$$f_{j00} = M_j \\ f_{j04} = f_{j40} = Q_j \\ f_{j44} = -P_j \quad (2)$$

The connexion between the parameters M_j , P_j and Q_j and m_j , q_j and p_j measured in M.K.S. units (p_j in kg) is:

$$c^2 M_j = 9km_j/13 \\ c^2 Q_j = (3/2)(26\pi\epsilon_0/k)^{-1/2} q_j \\ c^2 P_j = 9kp_j/13 \quad (3)$$

where k is Newton's constant of gravity and ϵ_0 is the permittivity constant for free space. It is to be noted also that (1) is a direct consequence of the theory and is in accord with the circumstance of attraction for masses of like sign and repulsion for charges of like sign.

In the limit of isolation, the right-hand side of the first equation of (1) would vanish since $(r_{ij} \rightarrow \infty)$ so that the vectors ξ_j satisfy the homogeneous wave equation in 0.4 space with constant coefficients, a situation compatible with the other equation of (1). In this limit it would be expected that the E.I.H. method would yield in the usual case:

$$M[(1 - \xi_{,0}^2)^{-1/2} \xi_{,0}]_{,0} = 0 \quad (4)$$

for the equation of motion when ξ is independent of x^4 . Since there is now no need to retain the subscript j , it

will be suppressed as in (4). A Lagrangian on which (4) may be based is:

$$L_0 = -M(1 - \xi_{,0}^2)^{1/2} \quad (5)$$

Now it is possible to surmise the Lagrangian for the case of dependence of ξ in x^4 by noting that equation (1) is invariant in form under rotations in 0.4 space so that the Lagrangian will involve the invariants $\eta^{AB} \xi_{,A} \xi_{,B}$ and $f^{AB} \xi_{,A} \xi_{,B}$. The latter observation combined with the requirement that aside from an additive constant the new Lagrangian reduce to equation (5) for $\xi_{,4} = 0$ yields via elementary considerations on functional relations:

$$L = f^{AB} \xi_{,A} \xi_{,B} (\eta^{AB} \xi_{,A} \xi_{,B})^{-1} [1 - (1 - \eta^{AB} \xi_{,A} \xi_{,B})^{1/2}] \quad (6)$$

The Lagrangian equation (5) precludes existence of velocities $> c$, since $\xi_{,0}^2 < 1$ ($x^0 = ct$). This does not seem to be the case for L given by equation (6) since reality requirements imply $\xi_{,0}^2 < 1 + \xi_{,4}^2$. Fortunately there exists a simple solution of the equations of motion implied by (6) consistent with $f^{AB} \xi_{,AB} = 0$ and $\eta^{AB} \xi_{,A} \xi_{,B} = \text{constant}$, namely:

$$\xi_1 = a_1 + b \cos \alpha \sin(\omega\chi^0 - k\chi^4) \\ \xi_2 = a_2 + b \sin \alpha \sin(\omega\chi^0 - k\chi^4) \\ \xi_3 = a_3 + b \cos(\omega\chi^0 - k\chi^4) \quad (7)$$

which corresponds to a 'particle' moving with angular velocity ωc in a circular path centred at (a_1, a_2, a_3) with radius b in the plane with azimuth angle $\varphi = \alpha$. The phase velocity in units of velocity of light on 0.4 space is given by:

$$V_p = w/k = Q/M \pm [Q^2/M^2 + P/M]^{1/2} \quad (8)$$

so that equations (7) and (8) together with reality requirements enable the establishment of the equivalent inequalities:

$$|wb| < |V_p|(V_p^2 - 1)^{-1/2} \\ |V_p| < |wb|(w^2b^2 - 1)^{-1/2} \quad (9)$$

$|wb|$ represents the magnitude of the peak velocity in units of c .

The assumption that the light variations are in some way correlated with the positions of the component 'particles' would yield some interesting relationships between the parameters describing the 'particles'. As an example, the size estimate $\sim 1,000$ light years in conjunction with ~ 10 year period would yield $|bw| = \sim 100$. Consequently, if P may be neglected in equation (8) and the plus sign is taken, equation (9) would yield an upper limit for Q/M of $\sim \frac{1}{2}$, which from equation (3) indicates that the upper bound of $|q/m|$ is $\sim 10^{-11}$ coul./kg and consequently that the ratio of electrical to gravitational forces between like particles is ~ 1 .

If the mass is of the order of the Sun's mass then one could surmise that the principal parts of 3C273 giving rise to the ~ 10 year fluctuation can have no more than $\sim 10^{28}$ electrons or protons so that the ratio of mass associated with charge to the total mass is $\sim 5 \times 10^{-23}$ for electrons and $\sim 10^{-19}$ for protons.

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¹ Greenstein, J., *Sci. Amer.*, **209**, 54 (1963). Fowler, W. A., *Rev. Mod. Phys.*, **38**, 545 (1964).

² Smith, H. J., and Hoffleit, D., *Nature*, **198**, 650 (1963).

³ Amer. Astro. Soc. Meeting, Montreal, December 29-31, 1964.

⁴ Gregory, C., *Nature*, **192**, 1320 (1961); *Phys. Rev.*, **125**, 2136 (1962).