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Although Watson was probably more widely known for his books, he produced a very large number of original papers on such topics as asymptotic expansions, Bessel functions, singular moduli, and problems arising out of the work of Ramanujan. He had a remarkable power of solving special problems. Much of his work was suggested by physical problems; but he was not really interested in the physical applications, being concerned only with the mathematical aspects of a problem.

While it is impossible in a short notice to deal adequately with Watson's original work, there are two papers of his which may, perhaps, be singled out for mention. One is that on 'general transforms', or, as they are sometimes known, Watson transforms. It has long been familiar that, under suitable conditions on f(x), the relation:

$$g(x) = \int_0^\infty k(xy) f(y) \, \mathrm{d}y \tag{1}$$

implies that:

$$f(x) = \int_{0}^{\infty} k(xy) g(y) \, \mathrm{d}y \tag{2}$$

when k(u) is either of the functions $\sqrt{(2/\pi)} \cos u$, $\sqrt{(2/\pi)} \sin u$. This raises the question of determining the most general function k(u) (satisfying appropriate restrictions) for which the same result holds. The problem was investigated independently in two papers, both published in 1933, one by Watson, and the other by Hardy and Titchmarsh. The two papers are complementary rather than overlapping. Without going into details, one can say roughly that Hardy and Titchmarsh required (1), (2) to hold in the sense of ordinary convergence, or some simple summability method, and therefore had to impose rather severe restrictions on the functions considered; whereas Watson required only that (1), (2) should hold in 663

some more general sense, and was therefore able to obtain results under much weaker restrictions.

One of Watson's last papers was on periodic sigma functions. The sigma function $\sigma(z)$ is defined by:

$$\frac{z^2 \log \sigma(z)}{\mathrm{d}z^2} = -p(z)$$

where p(z) is the Weierstrassian elliptic function; the two arbitrary constants implicit in this definition are fixed by specifying the behaviour of $\sigma(z)$ near the origin. The function $\sigma(z)$ differs from p(z) in not being doubly periodic; in place of the relations:

$$p(z+2\omega_1) = p(z+2\omega_2) = p(z)$$

(where $2\omega_1$, $2\omega_2$ is a pair of primitive periods), we have: $\sigma(z+2\omega_1) = -e^{2\eta_1(z+\omega_1)}\sigma(z);$

$$\sigma(z+2\omega_{z}) = -e^{2\eta_{z}(z+\omega_{z})}\sigma(z);$$

where η_1 , η_2 are constants depending on ω_1 , ω_2 . Elliptic and related functions had been so extensively investigated that one would not have imagined that any major problem connected with them remained to be considered. It was, however, left for Watson to point out that, while $\sigma(z)$ cannot be doubly periodic, it may be singly periodic. It is clear that this will happen if $\eta_1 = 0$, the period then being $4\omega_1$; Watson showed that any case in which $\sigma(z)$ is singly periodic can be reduced to this case by a suitable choice of the primitive periods. This is fairly easy; a much harder problem is that of determining in what cases (if any) the relation $\eta_1 = 0$ is, in fact, satisfied. Watson showed that $\eta_1 = 0$ if, and only if, the ratio ω_2/ω_1 takes one of an enumerably infinite set of values; and he investigated this set of values in some detail.

On his retirement, Watson expressed his intention of preparing a new and enlarged edition of *Modern Analysis*. It appears that this was never completed; and the world of mathematics is the poorer. B. KUTTNER

NEWS and VIEWS

U.S. National Academy of Sciences : Foreign Associates

THE following have been elected foreign associates of the U.S. National Academy of Sciences:

Prof. J. Brachet, professor and director of the Laboratoire de Morphologie Animale, Université Libre de Bruxelles. Prof. Brachet's work has been concerned with the distribution of two types of nucleic acid (DNA and RNA) in the cell and with interactions between the nuclear material of the cell and the surrounding cytoplasm.

Dr. J. M. Harrison, deputy to the Deputy Minister of Mines and Technical Surveys, Ottawa, Canada, and former director, Geological Survey of Canada. Dr. Harrison has made notable contributions to the understanding of the geology of the Canadian shield, particularly in working out the relation of ore deposits to complex Precambrian structures.

Prof. W. R. Hawthorne, Hopkinson and Imperial Chemical Industries professor of applied thermodynamics, University of Cambridge. Prof. Hawthorne's major contributions have been in connexion with aircraft propulsion, particularly in the development of turbo-type engines and related fundamentals of fluid mechanics and combustion.

Dr. P. B. Medawar, director, National Institute for Medical Research, Medical Research Council, Great Britain. Dr. Medawar's work has been concerned with tissue transplantation, including the demonstration of the phenomenon of acquired immunological tolerance in experimental animals for which he was awarded the 1960 Nobel Prize for Physiology and Medicine. Prof. Jean Leray, professor, Collège de France, Paris. To further the field of partial differential equations, Prof. Leray has delved deeply into algebraic topology and related fields, introducing new concepts and methods which have been a major factor in revolutionizing parts of homotopy theory, algebraic geometry, and modern analysis.

Dr. Sin-Itiro Tomonaga, president of the Science Council of Japan. Dr. Tomonaga has been a leader in the active and productive development of the field of theoretical physics in Japan and, in addition to his own research on nuclear forces, quantum field theory, cosmic rays, and many-body problems, has played an important part in the over-all development of Japanese science and education.

New Members

Prof. R. A. Alberty, professor of chemistry, University of Wisconsin; Dr. J. G. Baker, research associate, Harvard College Observatory, and consultant in optical physics; Prof. R. H. Bing, professor of mathematics, University of Wisconsin; Prof. D. H. Blackwell, professor of statistics, University of California, Berkeley; Prof. G. H. Büchi, professor of organic chemistry, Massachusetts Institute of Technology; Dr. J. W. Chamberlain, associate director, Space Division, Kitt Peak National Observatory; Prof. Erwin Chargaff, professor of biochemistry, Columbia University College of Physicians and Surgeons; Prof. R. F. Christy, professor of physics, California Institute of Technology; Prof. V. G. Dethier, professor of zoology and psychology, and associate, Neurological Institute,