both amplitude and phase. An image of the original object can then be observed in the appropriate region of the diffraction pattern of the half-tone mask. The image using conventional illumination is shown in *b* and the elongation of the peaks due to the wave-length spread is clearly visible. The considerable improvement in resolution when the laser is used as the source is shown in *c*.

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Hysteresis and Grain Size Effects in the Superconducting Phase Transition

FOR specimens of superconductors the resistance as a function of temperature or magnetic field is of the form shown in Fig. 1[1–3]. Baldwin[2], using thin-film specimens, has shown that minor loops of the type shown dotted can be followed and the cycle can be halted at any point, so that there must be stable or metastable states corresponding to all points on the curves. Faber[4] has shown that superheating effects are small and non-reproducible. This communication proposes a cell model which gives a simple qualitative explanation of these results.

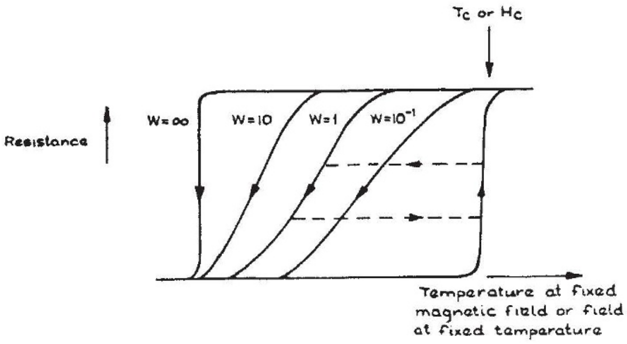

Fig. 1. The resistance of a superconductor as a function of temperature or magnetic field for various values of grain size *w*

If the change of surface free energy at the cell boundary on converting the cell from normal to superconducting is less than the free energy per unit area of a superconducting–normal interface in a cell, then the free energy to form a stable superconducting nucleus (and thus make the whole cell superconducting) decreases as the curvature of the cell walls increases.

As a simple example consider a system of cells of material which can initially be regarded as being spherical and completely isolated from one another by either chemical or crystallographic barriers. Fig. 2 shows part of one cell in which the change from the superconducting to normal state is taking place by nucleation at the cell wall. If the free energy of the cell wall is independent of the state of the material and there is a surface free energy γ_1 at the superconducting–normal interface, then if the decrease in volume-free energy on changing state is G_v, it is possible to write down the change of free energy required to produce a nucleus. Maximizing this expression gives the size of the just stable nucleus and the free energy required to create it. For the case shown on Fig. 2 this is:

$$G_1^c = 2\pi\gamma\left[AR^3 + \frac{2R}{A} + 2R^2 + \frac{1}{A^2} - \left(R^2 + \frac{1}{A^2}\right)^{\frac{1}{2}}\left(AR^2 + 2R + \frac{1}{A}\right)\right] \quad (1)$$

where $A = G_v/\gamma_1$ and hence G^x increases with R.

For large values of R, there is the possibility of transformation by the formation of hemispherical nuclei. The surface-free energy (γ_2) in this case will be larger than γ_1 due to the disturbance of the magnetic field near the interface. The free energy for such a nucleus is:

$$G_2^c = \frac{8\pi\gamma_2^3}{3G_v^2} \quad (2)$$

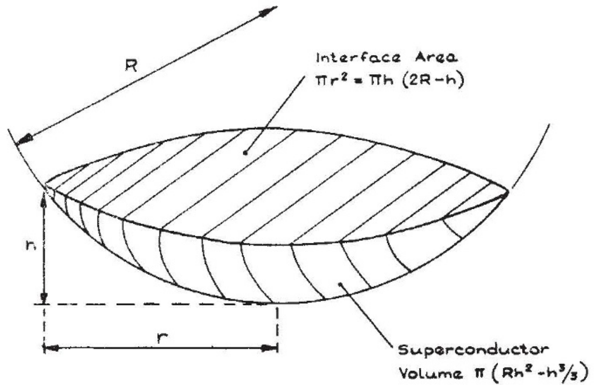

Fig. 2. Part of cell transforming from the normal to the superconducting state

Values of G_v and γ can be obtained from the normally accepted theory[5] and it can be shown that $\gamma \to 0$ at the transition point which accounts for the absence of super-heating phenomena.

If the value of the free energy G^x is large the probability $\exp(-G^x/kT)$ of the cell transforming becomes very small and cells will remain in a (metastable) normal state. Since there will be a distribution of cell sizes (or in the maximum curvatures in cells), as a specimen is supercooled, more cells will convert giving resistance curves of the form shown in Fig. 1, the differences between the curves being accounted for by the differences in the number of larger cells.

The cells need not be completely isolated. If one of a pair of cells with a circular area of contact of radius r is superconducting, the other will only become so by infection if r is greater than some critical value r^x. If the walls around the contact area are plane with the assumptions made previously this is given by:

$$r^x = \frac{2\gamma_2}{G_v} \quad (3)$$

Hence the cell boundaries need not be continuous provided that any gaps are less than a critical size. The cell walls could thus be composed of intersections of grain boundaries, non-metallic inclusions or even grains of completely different orientation. The concentration of such defects will almost certainly increase with decreasing grain size giving smaller cells with easier nucleation, thus explaining the grain size effect.

A more elegant treatment of this type of model could be made by the method used by Cahn and Hilliard[6,7] for small nuclei. Such a treatment would remove difficulties associated with having cell sizes comparable to the coherence length. At low super-saturations such a treatment should give similar results to that used here which is more easily visualized and gives a qualitative explanation of the results found.

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