Variation in the Ratio of Strontium-89 to Strontium-90 in Precipitation Samples with Sampling Height

It was shown that during March–December 1959 (ref. 1) the specific activities of fission products from fall-out in precipitation increased with sampling height. Some evidence was also found for an increase in the ratio $^{89}Sr/^{90}Sr$ with sampling height, but this increase was not significant ($P \sim 90$ per cent).

Table 1. Ratio of Strontium-89 to Strontium-90 in Precipitation Collected at Eight Different Heights in the Period October, 1961–September, 1963

Height (m) Month	1	7	23	39	56	72	96	123	Mean	Mm precipitation and group
Oct. 61	127·0	93·9	117·0	75·3	73·2	90·8	82·7	72·0	78·3	70 II*w*
Nov. 61	112·0	106·0	85·9	97·1	106·0	112·0	120·0	108·0	105·9	48 II*w*
Dec. 61	65·3	64·0	59·5	42·5	29·1	46·6	71·5	61·7	55·0	59 I*w*
Jan. 62	57·5	44·3	53·2	50·6	52·9	54·4	16·2	51·8	47·6	70 I*w*
Feb. 62	36·9	25·9	30·7	36·3	(29·5)	28·8	31·1	33·5	31·6	56 II*w*
Mar. 62	8·9	10·7	14·9	11·3	15·2	11·3	14·4	(10·7)	12·2	24 I*d*
Apr. 62	8·5	11·0	11·9	9·1	13·1	12·7	27·2	13·8	13·4	20 I*d*
May 62	9·4	10·2	(10·1)	8·2	9·5	8·7	10·4	8·7	9·4	57 II*w*
June 62	4·1	6·4	3·7	1·3	6·1	7·2	8·6	2·1	4·9	28 I*w*
July 62	4·2	5·1	5·5	3·8	4·8	6·5	5·5	4·2	5·0	112 II*w*
Aug. 62	7·9	5·6	6·4	3·9	3·1	6·8	4·7	3·6	5·3	111 I*w*
Sep. 62	19·0	17·5	18·6	14·2	14·1	16·7	20·1	14·3	16·8	36 II*w*
Oct. 62	22·2	22·1	17·6	17·3	16·6	18·2	20·2	19·7	19·2	25 II*w*
Nov. 62	45·3	29·3	41·2	38·4	42·1	36·2	41·2	36·1	38·7	30 II*w*
Dec. 62	20·6	10·9	14·2	24·9	19·6	(19·1)	16·3	(18·2)	18·0	20 I*d*
Jan. 63 Feb. 63	12·1	11·4	13·3	15·6	19·5	20·8	20·1	19·3	16·5	4 II*d* 7
Mar. 63	16·0	14·4	17·3	15·9	16·4	17·6	15·7	16·5	16·2	15 II*d*
Apr. 63	11·9	10·2	11·6	11·9	10·5	11·4	11·0	10·4	11·1	37 II*w*
May 63	7·0	7·5	7·2	7·6	6·7	7·8	7·4	7·8	7·4	22 II*d*
June 63	5·2	5·3	5·5	5·4	5·2	5·4	6·6	(5·8)	5·6	35 II*w*
July 63	3·8	4·3	4·4	4·0	4·1	4·6	4·3	4·1	4·2	73 II*w*
Aug. 63	2·6	1·8	2·7	2·5	2·5	2·5	2·9	2·5	2·5	123 II*w*
Sep. 63	1·3	0·9	0·8	1·3	1·4	1·8	1·4	1·5	1·3	24 I*d*

Figures in brackets are missing values calculated by the method of least squares. These figures are omitted in the regression analysis.

Table 2. Comparison of the Lines in Group I Analysis of Variance

Variation	Sum of squares of deviations	Degrees of freedom	Variance	Test
about lines	0·4147	57	$0{\cdot}7275 \times 10^{-2}$	
between slopes	$0{\cdot}5045 \times 10^{-1}$	1	$0{\cdot}5045 \times 10^{-1}$	6·94

Table 3. Comparison of the Lines in Group II. Analysis of Variance

Variation	Sum of squares of deviations	Degrees of freedom	Variance	Test
about lines	$0{\cdot}7378 \times 10^{-1}$	114	$0{\cdot}6472 \times 10^{-3}$	
between slopes	$0{\cdot}4952 \times 10^{-2}$	1	$0{\cdot}4952 \times 10^{-2}$	7·65

After the resumption of large-scale nuclear testing in September 1961, it became possible to repeat the experiment over a longer period. The $^{89}Sr/^{90}Sr$ ratios were determined in precipitation collected at eight different heights in the meteorological mast at Risö after the principles used in 1959 (ref. 1).

Table 1 shows the results for the 24-month period: October 1961–September 1963.

The $^{89}Sr/^{90}Sr$ ratios in Table 1 were treated as follows. The ratios were converted to the logarithms, because the figures were logarithmically distributed. The converted ratios for each month were divided by the corresponding (converted) mean ratio for the month. The purpose of this operation was to eliminate the effect of the time variation of the $^{89}Sr/^{90}Sr$ ratios. The months were separated into two groups, one with high variance (I) and one with low (II). This was necessary for the performance of a regression analysis, where the differences in the variances have to be insignificant. The two groups of months were each sub-divided into two sub-groups, one representing 'dry' month (*d*), that is, months with less than 25 mm precipitation and another representing 'wet' month (*w*), that is, months with 25 mm and more precipitation (Table 1).

The following regression lines were calculated:

$$y = 1 + 0{\cdot}1040 \times 10^{-2}\,(x - 46{\cdot}55) \qquad (\mathrm{I}d)$$

$$y = 1 - 0{\cdot}4452 \times 10^{-3}\,(x - 52{\cdot}13) \qquad (\mathrm{I}w)$$

$$y = 1 + 0{\cdot}4050 \times 10^{-3}\,(x - 52{\cdot}13) \qquad (\mathrm{II}d)$$

$$y = 1 + 0{\cdot}6857 \times 10^{-5}\,(x - 52{\cdot}39) \qquad (\mathrm{II}w)$$

(where y is the transformed $^{89}Sr/^{90}Sr$ ratio, and x is the sampling height).

An analysis of variance showed significant differences between the slopes for dry and wet months for both group I ($P \sim 99$ per cent) and group II ($P > 99$ per cent) (Tables 2 and 3).

Hence, it is evident that in 'dry' months the slope was significantly steeper than in the 'wet' months, that is, the $^{89}Sr/^{90}Sr$ ratios in 'dry' months were significantly higher in the upper precipitation collectors than in the lower. An explanation for this phenomenon has been proposed earlier[1]: it is assumed that the dry deposit, which yields the higher concentrations in the upper precipitation collectors, is of greater particle-size than the deposit from rain. Larger and heavier particles will be deposited more readily than fine particles, and consequently the coarse particles will consist of younger fission products (greater $^{89}Sr/^{90}Sr$ ratio) than the fine ones.

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[1] Aarkrog, A., *Nature*, **188**, 482 (1960).

Equivalence of the Schroedinger and Heisenberg Pictures

In a recent re-examination[1] of the foundations of quantum mechanics, Dirac asserts, *inter alia*, that the Heisenberg and Schroedinger pictures in quantum electrodynamics are not equivalent because the unitary operator which connects both pictures, namely $e^{iHt/\hbar}$, where $H = H_{\text{Dirac field}} + H_{\text{Maxwell field}} + H_{\text{interaction}}$, is the total Hamiltonian in the Schroedinger picture, does not exist. It is the purpose of this communication to question this assertion.

It is assumed from the outset that H is, in common with all observables in quantum mechanics, self-adjoint; that is, $H = H^{\dagger}$ and $\mathfrak{D}_H = \mathfrak{D}_{H^{\dagger}}$, where $\mathfrak{D}_H$ and $\mathfrak{D}_{H^{\dagger}}$ are the domains of H and its adjoint in the Hilbert space $\mathfrak{H}$ of the system. Accordingly, if $\{E(\lambda)\}$ are the projectors comprising the resolution of the identity belonging to H and if $F(\lambda)$ is an arbitrary complex-valued function of λ, then the operator $F(H)$, with the domain $\mathfrak{D}_F$, is defined[2] by the Radon–Stieltjes integral: