LETTERS TO THE EDITOR

PHYSICS

Theory of Striations in Gas Discharges

So far, a satisfactory theory for the well-known phenomenon of striations in gas discharges has proved elusive[1]. This communication delineates the basic elements of a theory for the stationary striation with a fuller account to be offered later encompassing the moving variety as well[2]. The central mechanism has been provided in relation to the manifestation of Liesegang rings[3], which represents the underlying regularity producing repetitions of nodes and antinodes proposed earlier by Morgan[4]. While the two physical phenomena entail entirely different processes, it happens that the analytical formalisms are very much alike. Indeed, the non-stationary striation has no equivalent in the formation of periodic precipitates, involving as it does much more complex processes.

A signal outcome of the present theory is the explanation of Goldstein's law[5] relating the spacings of the stationary striations l with the ambient pressure p:

$$l \propto p^{-m} \tag{1}$$

where m is very nearly 1/2. In an analogous development of the governing Poisson equation as for the Liesegang phenomenon[3], it can be shown that the spatially periodic behaviour of the potential as well as the electron distribution entail the vital Debye-length λ_D so that the spacing between striations obeys:

$$l \propto \lambda_D, \ \lambda_D^2 = \frac{kT}{4\pi n q^2} \tag{2}$$

where all quantities have their usual meaning. Hence, it is more sensible to associate the distance between striations with the carrier density n, that is, $l \propto n^{-1/2}$. The temperature dependency is somewhat involved for partial ionization but can be described in terms of the Saha equation; for complete ionization a $T^{1/2}$ dependence arises.

The connexion between l and the pressure itself is not as direct since it very much depends on the nature of the gas and its degree of ionization. For simple gases like hydrogen, it may be seen that:

$$P = (n_n + 2n)kT \tag{3}$$

where n_n denotes the number of neutral atoms per unit volume and $n = n_i = n_e$, n_i and n_e being the related concentration of the ions and electrons, respectively. In the limit of a strongly ionized system, $n_n \to 0$ and P is just proportional to n, whereby it follows that $m \sim 1/2$, but not precisely so for departures from complete ionization.

Since striations have been customarily observed in weakly ionized gases, it may be well to assess the question of incomplete ionization more fully. For simple systems such as:

$$A \to A^+ + e \tag{4}$$

let N be the total neutral particle density which admits:

$$n = \frac{P}{kT} - N \tag{5}$$

From equation (2), it follows that:

$$\lambda_D^2 = \frac{kT}{4\pi q^2 \dfrac{P}{kT}\left(1 - \dfrac{NkT}{P}\right)}$$

$$= \frac{(kT)^2}{4\pi q^2 \,.\, P}\left(1 - \frac{P_0}{P}\right)^{-1}, \quad P_0 = NkT \tag{6}$$

Since $1 > P_0/P > 1/2$ over the entire range of ionization:

$$l \propto \lambda_D \simeq \frac{kT}{2\sqrt{\pi}\, q P^{1/2}\left(1 - \dfrac{1}{2}\dfrac{P_0}{P}\right)} \tag{7}$$

which accounts for slight departures of m from 1/2 even for weakly ionized gases; the formula has no meaning for $P = P_0$.

Clearly, it is far simpler to relate the spacings of the striations directly with the electron density, and in this regard the spacings can be either too large or too small for visual observation. In discharges having an ambient temperature somewhat below 1,000° K, an electron density of $n = 10^{14}$/c.c. would have striations spaced a fraction of 1 mm apart. At $n = 10^{20}$/c.c., the expected separation would be about 10^{-7} cm. In any event, intermediate ranges of electron density are essential for ready detection of the striation phenomenon.

It has been tacitly supposed that the periodic luminous appearance of the discharge somehow reflects the spatial fluctuations of the electron density. Without necessarily entering into the details of the recombination process ultimately leading up to the emission of radiation, at some stage associated with the release of photons:

$$\frac{dn_i}{dt} = \frac{dn_e}{dt} = -kn_i n_e \tag{8}$$

Hence, since the photon production relates to an intensity

$$I \propto \frac{dn_i}{dt} \tag{9}$$

a periodic variation in I will result so long as n_e itself is periodic.

Perhaps it should be mentioned that the reaction rate equation (8) may not merely be representative of volume recombination but also of the radiative step in the three-body mechanism recently discussed by D'Angelo[6]. The latter appears more favourable than the dissociative recombination process formerly viewed as dominating over direct volume recombination.

A further interesting aspect is next noted regarding the distinctness of the striations in terms of the electron density. The presence of a large number of electrons tends to enhance light output while simultaneously providing a greater variation of intensity across the width of a striation. But as already mentioned, high concentrations of carriers produce a bunching or crowding of the striations. Thus the periodic glow of a discharge is optimizable.

As for the Liesegang phenomenon[3], non-linear effects may arise in which strict periodicity of the striations fails to occur. Our present description applies in the linear limit. The non-linearity will be covered in a more comprehensive development incorporating the non-stationary behaviour[2].

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1 Sodomsky, K. F., *J. App. Phys.*, **34**, 1860 (1963).
2 Aranow, R., and Gold, L. (to be published).
3 Gold, L., *Nature*, **202**, 889 (1964).
4 Morgan, G. D., *Nature*, **172**, 542 (1953).
5 Francis, G., *Handbuch Phys.*, **22**, 137 (Springer-Verlag, 1956).
6 D'Angelo, N., *Phys. Rev.*, **121**, 505 (1961).