LETTERS TO THE EDITOR

MATHEMATICS

Gravitational Collapse

In recent discussions[1] of the problem of gravitational collapse reference has repeatedly been made to the 'nucleonic mass' of a body as an integral involving the density and pressure of the material. In this, modern work has followed earlier results by Eddington[2] and Tolman on the supposed particle number integral for a spherically symmetrical configuration, according to general relativity.

It is the purpose of this communication to point out not just that the integral used is wrong, but also that no such integral can possibly exist. The distribution of density and pressure through a static configuration cannot describe the amount of energy released during condensation without full knowledge of the dependence of the internal energy of the material on the state variables. The original misunderstanding is probably due to the dangerous notion that, although general relativity is a continuum theory, an invariant integral related to mass must concern the invariant particle number.

Pre-relativistically it is well known that if any configuration is attained by homologous contraction from infinity with materials in which adiabatically the pressure varies like the 4/3rds power of the density, then no energy is released or taken up. With stronger dependence of pressure on density, energy is released and vice versa. Note that there is no requirement, relativistically or classically, for the sphere to be made up of uniform material; each spherical layer may have its own equation of state. Relativistically it is clear that before contraction the Schwarzschild m describes the rest-mass and hence the particle number, and will diminish during contraction if radiation is emitted.

We can now envisage two static condensations in which density and pressure are the same functions of a suitably defined radius. We can envisage one being made up out of layers of such materials that in the condensation no radiation was emitted, the other of materials releasing a great deal of energy during the process. Accordingly the initial m will have been the same as the final m for the first sphere but very much greater for the second. Thus, although in the final state any integral is the same for both spheres (because statically they are indistinguishable) the first sphere will be composed of fewer particles than the second one. Accordingly any attempt to deduce particle number from a relativistic integral over a static configuration is doomed to failure.

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[1] For example, Iben, I., *Astrophys. J.*, **138**, 1090 (1963).
[2] Eddington, A. S., *The Mathematical Theory of Relativity*, 122 (Cambridge, 1952).

RADIO ASTRONOMY

The Inverse Compton Effect as a Possible Cause of the X-ray Radiation of Solar Flares

The X-ray radiation of the Sun in the spectral range 60 Å $< \lambda <$ 0·1 Å has been investigated since 1948. A wealth of data has been obtained on the radiation of both the quiet Sun (60 $> \lambda >$ 5 Å) and of solar flares (down to $\lambda \sim$ 0·01 Å) (refs. 2–4). Theoretical calculations are based on the assumption that the X-ray radiation of the Sun is of thermal origin and may be caused by the recombination radiation, bremsstrahlung, and radiation in selected lines of a very hot and optically thin plasma. The most complete calculations have been made by Elwert[5,6].

However, the application of this theory to an analysis of the X-ray radiation of flares runs into major difficulties which may be stated as follows:

(*a*) For X-ray radiation of flares to be attributable to a thermal mechanism, there must be temperatures of the order of about 10^7 (ref. 4) or even 10^8 (ref. 6) and an electron density $n_e \sim 10^{10}$–10^{11} cm^{-3}, which appears unacceptable for many reasons.

(*b*) The spectrum of the X-ray radiation of flares in the energy range > 20 keV [1] cannot apparently be identified with a thermal spectrum. If the radiations of one and the same flare are treated in the framework of a thermal mechanism, different temperatures will have to be ascribed to different regions of the spectrum from 10^7 °K ($\lambda >$ 2 Å) to 10^8 °K ($\lambda <$ 0·5 Å). A more convenient expression for the spectrum of flares will be a power law: $F(\nu) \propto \nu^{-x}$, where, according to ref. 1, x = 2·5 and 4 (for two records, respectively).

(*c*) The origin of flat maximum in the spectral distribution of the X-ray radiation due to flares is not easily explicable[3,4].

(*d*) Nor can rapid variations in the fluxes of the X-ray radiation of flare with time be readily understood. In addition to this, the spectrum of this radiation changes appreciably during a flare[3].

Many more difficulties could be listed. In the circumstances described it is my belief that the X-ray radiation of solar flares ought to be explained by non-equilibrium processes. Most pronounced of them is the inverse Compton effect.

In collisions between the relativistic electrons with the energy $E \ll (mc^2)^2/\bar{\varepsilon}'$, and photons, the still harder photons, the average energy of which $\bar{\varepsilon} \approx \bar{\varepsilon}' (E/mc^2)^2 = \Delta E$ are produced, here $\bar{\varepsilon}'$ is the average energy of 'thermal' photons, ΔE is the average loss of energy by relativistic electrons during a single collision. Hence, if $\bar{\varepsilon}' \approx$ 2·5 eV and $\bar{\varepsilon} \sim 10^3$–$10^5$ eV, then E must be 10^7–10^8 eV. Due to the inverse Compton effect, the energy loss per second by relativistic electrons will be: $-(dE/dt)_c = \sigma_0\, cW(E/mc^2)^2 = 2 \times 10^{-14} \times W \times (E/mc^2)^2$, where W is the density of energy of radiation on the solar surface. The lifetime of a relativistic electron due to this energy loss at $W \sim$ 4 ergs/cm^3, $\bar{\varepsilon}'$ = 2·5 eV and $E = 10^7$ eV, will be approximately 2×10^5 sec.

From the foregoing it follows that the total energy of relativistic electrons inside the region of a flare must be about 10^{30} ergs. It is not at all unlikely that the $L\alpha$ quanta closed in the region of a flare have extremely high density which may be by one or two orders of magnitude greater than the density of all other photons. If so, the total energy of relativistic electrons may be several ten times lower.

The lifetime of the relativistic electrons closed in the region of a flare is determined by losses for ionization, and at $n_e = 10^{11}$ cm^{-3}, it will be about 300 sec. These losses cut off the energy spectrum of relativistic electrons from the low-energy side. This feature of the spectrum of relativistic electrons makes it possible to explain the observed flat maximum in the X-ray radiation of flares, if we take into account that there is a peak in energy distribution in the spectrum of thermal photons (Planck's function). The production of a fairly large number of relativistic particles during flares has been reliably estab-