

density of 16.55 logarithmic units. With the suggested choice of a zero point, it is not likely that we would be using negative logarithms very much. Indicators with a logarithmic response could use a simple linear scale on the indicating meter, and linear paper could be used on recorders. The indicator could be made to take care of varying gauge constants by incorporating a zero-suppression circuit.

With these advantages, there is one disadvantage of a logarithmic unit that must be admitted. In residual gas analysis it would not be convenient to calculate total densities from partial densities in logarithmic units. Whether or not this would be a serious drawback depends on how important such calculations are relative to the entire vacuum field. It might be desirable to have both a logarithmic and a linear unit of density. Whichever the choice, of either or both, it seems that the basic vacuum unit in high-vacuum work should be one of molecular density, rather than pressure.

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<sup>1</sup> Guthrie and Wakerling, *Vacuum Equipment and Techniques* (McGraw-Hill, 1948).

### Inhomogeneous Problems in Elasticity or Heat Conduction

N. LOUAT<sup>1</sup> has recently described a method of solving two-dimensional problems in elasticity by the replacement of free surfaces by two mutually independent arrays of edge dislocations chosen so as to produce zero stress at each free surface. M. A. JASWON<sup>2</sup> has described a similar procedure in which prescribed boundary displacements may be achieved by means of boundary distributions of point forces chosen to satisfy a set of Fredholm integral equations. The attraction of each method lies in the fact that the effect of a point force within an infinite medium may be calculated from a simple formula. Furthermore, the use of digital computer techniques allows the superposition of a large number of such forces.

It may be worth while to note that the technique of summation of the effect of point forces is also a useful one for treatment of inhomogeneous problems in elasticity or heat conduction. Such problems, although of considerable practical importance, have received little attention in the literature.

Suppose that a solution of the equations of elasticity has been obtained for homogeneous elastic material subject to given boundary conditions, boundary forces, and body forces. Let  $v_1, v_2, v_3$  denote the components of the displacement at a general point, let  $e_{ij}$  denote  $\frac{1}{2}(\partial v_i/\partial x_j + \partial v_j/\partial x_i)$ , and let  $\theta$  denote  $e_{11} + e_{22} + e_{33}$ . It may be shown that if the elastic parameters  $\lambda$  and  $\mu$  suffer a small change from constant values of  $\lambda_0$  and  $\mu_0$  to become:  $\lambda = \lambda_0 + l(x_1, x_2, x_3)$ ,  $\mu = \mu_0 + m(x_1, x_2, x_3)$ , then each displacement  $v_1, v_2, v_3$  is increased by an amount  $w_1, w_2, w_3$  which may be found as the result of applying to the homogeneous material an internal force distribution of components<sup>3</sup>:

$$g_i = 2\sum_j \partial(m e_{ij})/\partial x_j + \partial(\theta)/\partial x_i \quad (1)$$

The resulting stress distribution may readily be found in terms of the total displacement  $v_i + w_i$ .

The form of equation (1) is interesting in that it may be expressed as a sum of some terms such as  $e_{ij} \partial m/\partial x_j$  which become large when  $m$  changes abruptly, and some terms such as  $m \partial e_{ij}/\partial x_j$  which become large whenever  $m$  differs from zero in a region of high stress. Thus, if an elastic material contains intrusions of material of different elastic constants the effect of the intrusions is equivalent to a set of forces distributed over their surfaces and another

set distributed throughout their volume. In many instances this second set of forces is small so that the intrusions may be replaced by forces on their surfaces.

The analysis outlined above is not restricted to two-dimensional problems.

A similar treatment may be given of the equation of heat conduction through inhomogeneous material. The equation of heat conduction in a solid is:

$$\nabla k \cdot \nabla T + k \nabla^2 T = g \partial T/\partial t \quad (2)$$

where  $T$  is the temperature,  $k$  is the conductivity, and  $g = \rho c$ , where  $\rho$  is the density and  $c$  is the specific heat. Suppose that for constant parameters  $k_0$  and  $g_0$  a solution is  $T_0(x, y, z, t)$ . If the parameters  $k$  and  $g$  change to become:  $k = k_0 + A k_1(x, y, z)$ ,  $g = g_0 + A g_1(x, y, z)$ , then equation (2) may be satisfied by a solution  $T(x, y, z, t)$  of the form:

$$T(x, y, z, t) = \sum_{n=0}^{\infty} A^n T_n(x, y, z, t)$$

where the  $T_n$  are found in succession as solutions of the equations:

$$k_0 \nabla^2 T_n - g_0 \partial T_n/\partial t = g_1 \partial T_{n-1}/\partial t - \nabla(k_1 \nabla T_{n-1}) \quad (3)$$

The constant  $A$  is chosen to specify the scale of the inhomogeneities and has no other physical significance.

Each of equations (3) represents a temperature distribution  $T_n$  in a homogeneous material that contains a distribution of heat sources or sinks corresponding to the terms on the right-hand side. Thus, the temperature distribution within inhomogeneous material may be investigated in terms of that within homogeneous material containing a suitable distribution of heat sources.

An advantage of the form of equations (1) and (3) is that it is not necessary to choose a co-ordinate system to fit the shape of the inhomogeneities; thus the technique may be applied to treat the effect of irregularities of general shape. Some preliminary investigation has shown that there are many instances where use of equations (3) leads to a solution the terms of which may be found by algebraic recurrence relations. The technique is now being applied to several problems of practical interest.

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<sup>1</sup> Louat, N., *Nature*, **196**, 1081 (1962).

<sup>2</sup> Jaswon, M. A., *Nature*, **198**, 572 (1963).

<sup>3</sup> Heaps, H. S., *Trans. Amer. Geophys. Union*, **37**, 477 (1956).

### Use of Long Cavities for Aqueous Samples in an Electron Spin Resonance Spectrometer

In a comment on the electron spin resonance spectrometer designed especially for aqueous samples which has been described recently by Cook and Mallard<sup>1</sup>, Stoodley<sup>2</sup> has calculated that the method of using a multimode microwave cavity tends to degrade the sensitivity rather than improve it. I would like to point out that support for Stoodley's result is provided by a spectrum recorded by Varian Associates on their aqueous sample spectrometer and published in some recent advertising material<sup>3</sup>. The signal-to-noise for this spectrum is some thirty times better than for that presented by Cook and Mallard. Further, although the sensitivity was said to be comparable with that obtained by Commoner *et al.*<sup>4</sup>, in fact those workers claim a figure of  $2.5$  to  $5 \times 10^{-7}$  molar of  $Mn^{++}$  while that of Cook and Mallard is  $3 \times 10^{-6}$  molar. The sensitivity of the Varian spectrometer expressed in those terms is  $1 \times 10^{-7}$  molar. The difference in sensitivity between that achieved by Cook and Mallard and by Varian can be attributed to three causes. First, sensitivity is proportional to the square root of the microwave power and, while Varian quote 180 mW, Cook and Mallard appear to be using a 723 A/B klystron which gives 20 mW. Hence the sensitivity drops by a factor of three. Secondly,