This work was supported in part by funds provided under grant No. *DA-ARO(D)*-31-124-*G*124 from the U.S. Army Research Office (Durham).

D. A. McQuarrie

Department of Chemistry,
Michigan State University,
East Lansing,
Michigan.

A. T. Bharucha-Reid

Department of Mathematics,
Wayne State University,
Detroit,
Michigan.

[1] Bharucha-Reid, A. T., *Elements of the Theory of Markov Processes and Their Applications*, 73 (McGraw-Hill, New York, 1960).
[2] Erdélyi, A., *et al.*, *Tables of Integral Transforms*, 1, 238 (McGraw-Hill, New York, 1954).
[3] Singer, K., *J. Roy. Statist. Soc.*, Ser. B, **15**, 92 (1953).

PHYSICS

Sources of Harmonics of Low Order in the External Gravity Field of the Earth

The external gravitational potential, V, of the Earth at a point having the spherical polar co-ordinates, radius vector r, co-latitude θ and longitude λ, with respect to the centre of mass of the Earth, may be written in the form:

$$V = -\frac{GM}{r}\left\{1 - \sum_{n=2}^{\infty} J_n\left(\frac{R}{r}\right)^n P_n(\cos\theta) + \text{terms varying with longitude}\right\}$$

Here G is the constant of gravitation, M is the mass of the Earth, R is a scale factor, commonly the equatorial radius of the Earth, and $P_n(\cos\theta)$ is the Legendre polynomial of degree n.

King-Hele, Cook and Rees[1] have recently determined the coefficients of the harmonics of even order up to J_{12} from the secular changes in the nodes of orbits of seven artificial satellites, while coefficients of the harmonics of odd order up to J_9 have been determined from periodic changes in the orbits. Beyond J_4, the coefficients are of order 5×10^{-7}.

There are now sufficient data to enable some bounds to be placed on the regions within which the sources of these harmonics must lie. A repetition[2] of a calculation originally due to W. H. Munk and G. F. C. MacDonald shows that there is no relation between the coefficients of the harmonics so far determined and the coefficients of the development of the distribution of the continents in surface harmonics; accordingly the sources must be sought below the Mohorovičić discontinuity. An obvious possibility is that the sources may be irregularities in the boundary between the core and the mantle. Now in consequence of the well-known ambiguity in the inference of a density distribution from a gravity field, it is not possible to prove or disprove the existence of such irregularities from gravity data alone; we can, however, ask whether or not there are any irregularities, however great, at a particular depth that could produce the observed field, and if that is not possible, the sources must lie nearer the surface.

Suppose that the variation of density between radii r_1 and r_2 is given by $\sigma_n P_n(\cos\theta)$. Then the coefficient of P_n in the external field is:

$$J_n = -\frac{3}{(2n+1)(n+3)}\frac{\sigma_n}{\rho}\cdot\left(\frac{r_2}{R}\right)^{n+3}\left[1-\left(\frac{r_1}{r_2}\right)^{n+3}\right]$$

where ρ is the mean density of the Earth.

When n is of order 10 or more $(r_1/r_2)^{n+3}$ will be very small unless the two radii are nearly equal, so that the value of J_n is determined by the outer radius of the shell. Then:

$$J_n \simeq -\frac{3}{(2n+1)(n+3)}\frac{\sigma_n}{\rho}\cdot\left(\frac{r_2}{R}\right)^{n+3}$$

A reasonable value of σ_n/ρ is $\frac{1}{2}$.

If we take J_{12} to be 3×10^{-7},

$$\left(\frac{r_2}{R}\right)^{15} \text{ must be not less than } 7{\cdot}5 \times 10^{-5}$$

and hence $\frac{r_2}{R} > 0{\cdot}537$.

This limit is very close to the ratio of the radius of the core to the radius of the Earth, the value of which is 0·545, and further consideration makes it seem probable that whatever irregularities there may be in the core-mantle boundary, they are unlikely to produce the observed 12th order harmonic, the source of which must therefore be sought higher up in the mantle. Let t denote the difference $r_2 - r_1$, so that the formula for the coefficient J_n becomes:

$$J_n = -\frac{3}{2n+1}\cdot\frac{\sigma_n}{\rho}\cdot\frac{t}{R}\cdot\left(\frac{r_2}{R}\right)^{n+2}$$

Taking r_2/R to be 0·545, and σ_n/ρ to be $\frac{1}{2}$, t has to be 150 km, or about 1/20 of the radius of the core, to give the observed value of J_{12}. If such irregularities seem excessive (it is not clear whether present seismic data would reveal them) it follows that additional sources for the 12th and higher order harmonics must lie in the mantle. The value of J_{10}, -5×10^{-7}, requires t to be 65 km if the 10th order harmonic arises from irregularities in the core mantle boundary.

The conclusion appears to be that irregularities in the core-mantle boundary could produce the observed harmonics of order 10 and less, that the 12th order harmonic probably arises from irregularities above the core-mantle boundary and that higher harmonics certainly do. Evidently, then, determinations of the harmonics of order 13 and higher will be of great interest since they are the lowest that must unambiguously arise from density variations in the mantle.

The following list shows the harmonics for which the sources must probably be sought above different levels in the Earth:

Level	r/R	Order of harmonics which must have sources above the level
Boundary of inner core	0·216	> 4
Core-mantle boundary	0·545	> 12
Low velocity layer	0·95	> 113

This communication is published with the permission of the Director of the National Physical Laboratory.

A. H. Cook

Standards Division,
National Physical Laboratory,
Teddington, Middlesex.

[1] King-Hele, D. G., Cook, G. E., and Rees, J. M., *Nature*, **197**, 785 (1963).
[2] Cook, A. H. (to be published in *Space Sci. Revs.*)

Measurement of Relativistic Time Dilatation using the Mössbauer Effect

The Mössbauer effect allows of some of the most direct tests of certain simple relativistic predictions, and in this connexion we wish to report on the progress of some experiments similar in principle to those first reported by Hay, Schiffer, Cranshaw and Egelstaff[1,2]. These authors measured the relativistic frequency shift between a cobalt-57 source of 14·4-keV γ-radiation, near the centre of a rotating disk, and a resonant iron-57 absorber around the periphery of the disk. On spinning the disk the resonance absorption was found to decrease, giving increases in transmission of up to 6 per cent. This was shown to correspond to a relative frequency shift the