

Table 2. EFFECT OF SUCCESSIVE ALCOHOL WASHINGS ON POSITIVE AND NEGATIVE CHARGE AT pH 4 (SOIL 3)

No. of 10 ml. portions of 80% ethanol	Negative charge me %	Positive charge me %	Loss of charge in ethanol wash	
			Negative me %	Positive me %
0*	4.6	5.0	—	—
2	2.8	3.0	1.8	2.0
5	2.0	2.0	0.8	1.0
10	1.6	1.6	0.4	0.4
20	1.2	1.3	0.4	0.3
30	1.0	1.0	0.2	0.3
40	0.8	0.8	0.2	0.2
50	0.7	0.8	0.1	0.0
60	0.7	0.7	0.0	0.1

\* Measured by Schofield's method.

negative charges. There is a continual loss of positive and negative charge in roughly equivalent amounts until about 50 x 10 ml. portions of alcohol have passed through the sample. It is interesting to note that soil 3 is iso-electric in the vicinity of pH 4.

The explanation for the loss of charge in equivalent amounts is that the alcohol washing causes a decrease in electrolyte concentration with a resultant expansion of the double layers surrounding positive and negative charges. Under such conditions these double layers will overlap, resulting in the mutual neutralization of positive and negative charges with the loss of the cations and anions previously held as salt.

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<sup>1</sup> Sumner, M. E. (in the press).

<sup>2</sup> Schofield, R. K., *J. Soil Sci.*, 1, 1 (1949).

### Occurrence of Kaolinite in Association with Iron-pan

KAOLINITE has generally been recognized as the most widely occurring clay type in the humid tropics. Various theories in regard to the probable mode of occurrence of this clay type have been advanced from time to time. Prescott and Pendleton<sup>1</sup> have observed that in a region of high rainfall and under heavy leaching: "Kaolinite is likely to be most characteristic". They have also expressed the likelihood of kaolinite formation as a result of resiliification of gibbsite. According to Russell<sup>2</sup> kaolinites are formed "under conditions of a low concentration of bases", that is, in well-drained areas. It has also been suggested by Russell<sup>2</sup> that in Malabar laterite might have developed from a lithomarge (containing 30-50 per cent kaolinite with 15-40 per cent of iron oxides) under an indurated iron pan.

The observation reported in this communication seems to prove the latter view. Recently, while studying a soil profile on the slope of a hill (in Nsukka, Eastern Nigeria) a small specimen of indurated iron pan was encountered at a depth of about 3 ft. 3 in. from the ground surface. Within this specimen, some very fine white powdery material along with other soil materials were observed. The soil may broadly be classified as lateritic and the dominant vegetation is of savanna type. There are only two identifiable layers, namely, the upper surface layer, darker in colour due to organic matter, and the other layer, reddish in colour extending to a great depth. The iron-pan specimen was found in the second layer. Termites were observed to be very active in the close vicinity of the occurrence of the specimen. Due to a lack of necessary facilities for examining and analysing the soil materials, the specimen was dispatched to the Rothamsted Agricultural Experimental Station.

On the basis of the results of the X-ray analysis the white material in the specimen has been identified as kaolinite.

Further work is needed to explain the mode of occurrence of such iron pans in very freely drained materials.

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<sup>1</sup> Prescott, J. A., and Pendleton, R. L., *Laterite and Lateritic Soils*, 33 (Commonwealth Bur. Soil Sci. Tech. Comm., No. 47).

<sup>2</sup> Russell, E. J., *Soil Conditions and Plant Growth*, 556, 560 (Longmans, Green and Co., Ltd., London, 1961).

### STATISTICS

#### Generalized Inverse of a Singular Matrix

THE problem of 'inverting' singular matrices is by no means uncommon in statistical analysis. Rao<sup>1</sup> has shown in a lemma that a generalized inverse (*g*-inverse) always exists, although in the case of a singular matrix it may not be unique.

A technique having its basis in the following theorem may prove useful in the solution of many statistical problems, particularly those connected with the analysis of variance (Koop, J. C., unpublished communication).

Let  $X = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$  be a singular matrix in which the square sub-matrices *A* and *D* are non-singular and are such that the elements of  $A^{-1}BD^{-1}C$  and  $D^{-1}CA^{-1}B$  are close to zero. Then a *g*-inverse of *X* is approximated by

$$X_0 = \begin{bmatrix} A^{-1} & 0 \\ 0 & D^{-1} \end{bmatrix} \begin{bmatrix} A & -B \\ -C & D \end{bmatrix} \begin{bmatrix} A^{-1} & 0 \\ 0 & D^{-1} \end{bmatrix}$$

The proof of this theorem depends on Rao's lemma. If *X*- is a true *g*-inverse then according to this lemma  $XX-X = X$ . Thus if  $X_0$  approximates *X*- then the relation

$$XX_0X \approx X \tag{1}$$

will hold subject to the conditions stated in the above theorem. Now:

$$\begin{aligned} XX_0X &= X \begin{bmatrix} A^{-1} & 0 \\ 0 & D^{-1} \end{bmatrix} \begin{bmatrix} A & -B \\ -C & D \end{bmatrix} \begin{bmatrix} A^{-1} & 0 \\ 0 & D^{-1} \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \\ &= X \begin{bmatrix} I & -A^{-1}B \\ -D^{-1}C & I \end{bmatrix} \begin{bmatrix} I & A^{-1}B \\ D^{-1}C & I \end{bmatrix} \\ &= X + X \begin{bmatrix} -A^{-1}BD^{-1}C & 0 \\ 0 & -D^{-1}CA^{-1}B \end{bmatrix} \tag{2} \\ &\approx X \end{aligned}$$

since by hypothesis the displayed partitioned matrix has elements which are negligible. Hence  $X_0$  is an approximate *g*-inverse of *X*.

If it is desired to correct  $X_0$ , then we proceed as follows. Let  $X_0 = X^- + S$ , where *S* is a deviant matrix. If *S* can be determined then  $X_0$  can be corrected.

Now:

$$XX_0X = X(X^- + S)X = XX^-X + XSX = X + XSX \tag{3}$$

Since (2) and (3) must be identical:

$$SX = \begin{bmatrix} -A^{-1}BD^{-1}C & 0 \\ 0 & -D^{-1}CA^{-1}B \end{bmatrix} \tag{4}$$

Let  $S = \begin{bmatrix} \alpha & \beta \\ \theta & \mu \end{bmatrix}$  where  $\alpha, \beta, \theta$  and  $\mu$  are of the same order as the corresponding sub-matrices of  $X = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ . Premultiplying *X* by *S* and setting the resulting matrix expressions equal to the corresponding ones on the right-hand side of (4) and solving for  $\alpha, \beta, \theta$  and  $\mu$ , we find:

$$\begin{aligned} \alpha &= A^{-1}BD^{-1}C(BD^{-1}C - A)^{-1} \\ \beta &= -A^{-1}BD^{-1}C(BD^{-1}C - A)^{-1}BD^{-1} \\ \theta &= -D^{-1}CA^{-1}B(CA^{-1}B - D)^{-1}CA^{-1} \\ \mu &= D^{-1}CA^{-1}B(CA^{-1}B - D)^{-1} \end{aligned}$$