LETTERS TO THE EDITOR

ASTROPHYSICS

A New Method of directly measuring the Galactic Magnetic Field

WITH the announcement of the existence of polarization in the Crab Nebula at a wave-length of 21 cm[1], a new method of determining the galactic magnetic field becomes available—the measurement of the Faraday rotation of the plane of polarization due to the presence of anomalous dispersion in the neighbourhood of the 21-cm line of neutral hydrogen. This may be calculated as follows:

The index of refraction due to hydrogen atoms at frequency ν_0 is given by:

$$n = 1 + \frac{e^2 N(\nu_0) f}{4\pi m \nu} \frac{\nu_0 - \nu}{(\nu_0 - \nu)^2 + \left(\frac{\Gamma}{4\pi}\right)^2} \frac{1}{4}$$

where f is the Ladenburg f for the transition, $3 \cdot 8 \times 10^{-12}$. The factor 1/4 appears because only the atoms in the lower state cause a phase shift. Summing over all hydrogen atoms:

$$n - 1 = \frac{e^2 f}{16\pi m \nu} \int_{-\infty}^{\infty} \frac{(\nu_0 - \nu) N(\nu_0)}{(\nu_0 - \nu)^2 + \left(\frac{\Gamma}{4\pi}\right)^2} d\nu_0$$

When $\Gamma \to 0$ this integral becomes a Cauchy principal value.

$$n - 1 = \frac{e^2 f}{16\pi m \nu} \text{ c.p.v.} \int_{-\infty}^{\infty} \frac{N(\nu_0)}{\nu_0 - \nu} d\nu_0$$

A ray is thus delayed beyond a ray *in vacuo* by:

$$\Delta t = \int \frac{n - 1}{c} ds = \frac{ef}{16\pi m \nu c} \text{ c.p.v.} \int_{-\infty}^{\infty} \frac{\int N(\nu_0) ds}{\nu_0 - \nu} d\nu_0$$

which corresponds to a phase lag of:

$$\Delta\varphi = \omega \Delta t = \frac{e^2 f}{8mc} \text{ c.p.v.} \int_{-\infty}^{\infty} \frac{\int N(\nu_0) ds}{\nu_0 - \nu} d\nu_0$$

The optical depth $\tau(\nu_0)$ is given by:

$$\tau(\nu_0) = \frac{\nu}{c} \beta \int \frac{N(\nu_0)}{T_s} ds$$

where $\beta = 5 \cdot 45 \times 10^{-14}$ cm^3 sec^{-1} deg. K, and T_s is the spin temperature.

If we assume T_s constant:

$$\Delta\varphi = \frac{e^2 f}{8m\beta\nu} T_s \text{ c.p.v.} \int_{-\infty}^{\infty} \frac{\tau(\nu_0)}{\nu_0 - \nu} d\nu_0$$

The Faraday rotation ψ is the change in phase between the right and left circularly polarized components. If they are separated in resonant frequency by:

$$\Delta\nu = 2 \cdot 8 \times 10^6 B \text{ c.p.s.}$$

then

$$\psi = \frac{e^2 f}{8m\beta\nu} T_s \frac{d}{d\nu} \left(\text{c.p.v.} \int_{-\infty}^{\infty} \frac{\tau(\nu_0) d\nu_0}{\nu_0 - \nu} \right) \Delta\nu$$

numerically:

$$\psi = 6 \cdot 22\, T_s \frac{d}{d\nu} \left(\text{c.p.v.} \int_{-\infty}^{\infty} \frac{\tau(\nu_0) d\nu_0}{\nu_0 - \nu} \right) \Delta\nu$$

The absorption profile observed with a single paraboloid has been taken from Clark, Radhakrishnan and Wilson[2] to derive the frequency-dependent part of this expression. The resultant for a field of 10^{-6} gauss and $T_s = 100°$ K is plotted in Fig. 1. Some smoothing has inadvertently been introduced in the process of numerical integration. Since ψ is essentially dependent on the second derivative of optical depth with respect to frequency, the estimated rotation is somewhat sensitive to the band-width and saturation corrections to the measured absorption profile.

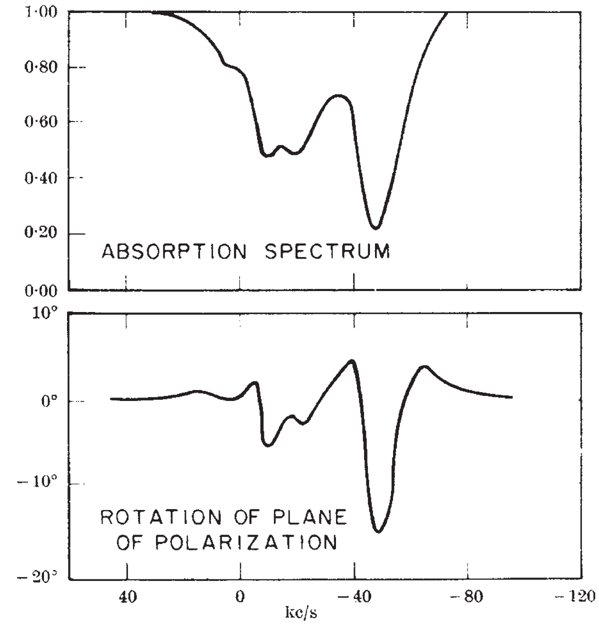

Fig. 1. Absorption profile and Faraday rotation in the Crab Nebula

The measurement of this rotation is in some ways not as difficult as an absolute measurement of the angle and amount of the polarization since instrumental polarization effects need not be considered in finding the frequency dependence of the polarization. Thus the question again reverts to having a sufficiently good signal to noise ratio to detect the polarization. For small magnetic fields the method is comparable in sensitivity with the more conventional measurement of Zeeman splitting in right and left circular polarizations.

This work was supported by the U.S. Office of Naval Research under Contract *Nonr* 220(19).

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[1] Radhakrishnan, V., and Morris, D., *Astrophys. J.*, **137** (in the press).
[2] Clark, B. G., Radhakrishnan, V., and Wilson, R. W., *Astrophys. J.*, **135**, 151 (1962).

RADIOPHYSICS

Radio Observations of Jupiter during 1962

IT has been suggested by Barrow[1] that a rigorous comparison of solar, geomagnetic and Jupiter activity in radio emission cannot be made until continuous observation of Jupiter is possible throughout an apparition. The results summarized in this communication will, it is hoped, help to fill some gaps in the available figures for the 1962 apparition.

Standard communications receivers, suitably modified, were used in conjunction with single folded dipole aerials