This equation is inapplicable below $\nu = 1 \cdot 24$. Below this diameter ratio, in any event, volume contractions can no longer be relied on to occur.

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[1] Bernal, J. D., *Nature*, **183**, 141 (1959).
[2] Mangelsdorf, P. C., and Washington, E. L., *Nature*, **187**, 930 (1960).
[3] Scott, G. D., *Nature*, **188**, 908 (1960).
[4] MacRae, J. C., and Gray, W. A., *Brit. J. App. Phys.*, **12**, 164 (1961).
[5] Parrish, J. R., *Nature*, **190**, 800 (1961).
[6] Rutgers, R., *Nature*, **193**, 465 (1962).
[7] Smalley, I. J., *Nature*, **194**, 1271 (1962).
[8] Mott, R. A., *Some Aspects of Fluid Flow*, Paper 14 (Edward Arnold, London, 1951).
[9] Denton, W. H., *A.E.R.E. E/R* 1095 (Harwell, 1953).
[10] Leva, M., *et al.*, *U.S. Bur. Mines Bull.*, **504**, 22 (1951).
[11] Oman, A. O., and Watson, K. M., *Refinery Management and Petroleum Chem. Tech.*, **36**, R-795 (1944).
[12] Ergun, S., and Orning, A. A., *Indust. Chem. Eng.*, **41**, 1179 (1949).
[13] Leva, M., *Fluidization*, 21 (McGraw-Hill, 1959).
[14] Happel, J., *Indust. Eng. Chem.*, **41**, 1161 (1949).

## Re-entrant Motion in Special Relativity

A RECENT communication[1] discusses the uniform rotation of a rigid circular disk as a problem in special relativity. Phipps is in error in supposing that there is more than one way to measure the peripheral velocity in a Lorentz frame in which the centre of the disk is at rest. He will see that the velocities $v$ and $r\omega$ are equal (if it is not at once obvious from the kinematics in $S$) by drawing a Minkowski diagram showing the approximately parallel world lines of adjacent markers. These move with velocity $v$ in $S$, and are at rest in $S^*$. $S^*$ ascribes a spatial displacement $\Delta x^*$ to the pair of events consisting of successive transits of a point fixed in $S$. Since the two markers are at rest in $S^*$, $\Delta x^*$ may instead be determined from a pair of events in their histories simultaneous in $S^*$. But then $\Delta x^*$ is just the rest displacement, namely, $2\pi r/n\sqrt{1-v^2/c^2}$. This reflects the well-known fact that in a rigid rotating (but non-holonomic) co-ordinate system the ratio circumference : radius is given by $2\pi : \sqrt{1-v^2/c^2}$. Substituting the above value for $\Delta x^*$ into the Lorentz formula, we find that the 'round-trip' velocity $r\omega$ and the relative velocity $v$ are in fact equal.

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[1] Phipps, T. E., *Nature*, **195**, 67 (1962).

I BELIEVE that Dr. Phipps is mistaken in his assertions (1) that the round trip velocity $v_{r.t.}$ of a point on the edge of a rotating disk, defined by $v_{r.t.} = \omega r = 2\pi r/T$, where $T$ is the period, is not equal to the instantaneous edge velocity, $v$, and (2) that $\omega r$ can become greater than $c$, the velocity of light.

Dr. Phipps's proof of these assertions depends on his assumption that a rapidly rotating disk remains rigid; whereas, independently of the strength of the disk, this cannot occur. Dr. Phipps's subsequent conclusions concerning time dilatation also depend on his incorrect proof. Therefore, there is no reason to reject the usual result of special relativity that a person riding on the edge of a rotating disk will age more slowly than his stationary twin.

Let us follow Dr. Phipps's derivation in detail. He considers a disk the outer edge of which is marked with point markers which divide it into $n$ segments, each of rest-length $s_0$. Let $n$ be sufficiently large so that the curvature of a segment can be neglected. Now consider a Lorentz frame $S^*$ in which a particular edge-segment of the disk is momentarily at rest as the segment passes a fiducial point in the laboratory system $S$.

Define two 'events' as the passage opposite the fiducial point in $S$ of two adjacent markers on the edge of the disk. These two events are separated by an interval ($\Delta x$, $\Delta t$) in $S$ and ($\Delta x^*$, $\Delta t^*$) in $S^*$, where, by special relativity:

$$\Delta x^* = \frac{\Delta x - v\,\Delta t}{\sqrt{1-v^2/c^2}} \tag{1}$$

Now, as noted by Dr. Phipps, $\Delta x = 0$ and $\Delta t = T/n$. Then, to obtain the instantaneous velocity $v$ from equation (1), it is only necessary to obtain $\Delta x^*$, which Dr. Phipps assumes is given by $\Delta x^* = s_0$. However, if the circumference of the disk is $2\pi r$ when measured in the laboratory system $S$, then by symmetry, the length of any segment measured in $S$ is $s_0 = 2\pi r/n$. Therefore, in the frame $S^*$, which is at rest with respect to one of the segments, this segment has the greater length $s_0/(\sqrt{1-v^2/c^2})$. This follows from special relativity, according to which any object appears shortened if it is moving; that is, it appears longest in its own rest system. Substituting:

$$\begin{aligned} &\Delta x^* = s_0/\sqrt{1-v^2/c^2} = 2\pi r/(n\sqrt{1-v^2/c^2}), \\ &\Delta x = 0,\ \Delta t = T/n \end{aligned} \tag{2}$$

in equation (1), we obtain the usual result:

$$v = 2\pi r/T = \omega r \tag{3}$$

in disagreement with the result obtained by Dr. Phipps.

The validity of this derivation depends on the circumference of the rotating disk remaining $2\pi r$ when measured in $S$, rather than appearing foreshortened to $2\pi r\sqrt{1-v^2/c^2}$ as it would be if the circumference were rigid. Thus, we have assumed that the circumference stretches as a result of the motion, each segment attaining the length $s_0/\sqrt{1-v^2/c^2}$ in its own rest system. If we instead force the circumference to be rigid, then the radius of the disk must shrink. It is impossible kinematically for the radius and circumference to remain rigid and for the disk to stay in one piece. If the disk were infinitely strong as well as infinitely rigid, it could not be made to rotate, as work could not be done against its infinite cohesive forces.

I conclude that the round-trip velocity and instantaneous velocity are equal and that both remain less than $c$. Also, to attain edge speeds approaching $c$, rather than using an ideal rigid disk, we should use one of ideal rubber with one-way stretch.

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[1] Phipps, T. E., *Nature*, **195**, 67 (1962).

OBJECTIONS to my analysis of re-entrant motion generally fall into one of two categories, (1) criticisms that accept the analysis within its terms of reference (namely, those of ordinary special relativity), but view the result as sufficiently ludicrous to refute one or