Generalized Hartley–Ross Unbiased Ratio-Type Estimator

RATIO method of estimation is frequently used in sample surveys to estimate the population mean of the variable under investigation. Several ratio-type estimates can be formed. All these estimates are unsatisfactory in the sense that they are biased. This difficulty was overcome by Hartley and Ross[1], who proposed an unbiased ratio-type estimator for uni-stage sampling designs. In practice, however, we are generally faced with multi-stage sampling designs. This communication gives a generalized form of Hartley–Ross unbiased ratio-type estimator for multi-stage designs. Let N = number of first-stage units in the population, M = number of second-stage units in each of the first-stage units, P = number of third-stage units in each of the second-stage units, $Yijk$ = the value of the variable under investigations for the k-th unit in the j-th second-stage unit of the i-th first-stage unit, and

$$\bar{Y}ij. = \frac{1}{P}\sum_{1}^{P} Yijk,\ \bar{Y}i.. = \frac{1}{MP}\sum_{1}^{M}\sum_{1}^{P} Yijk \text{ and}$$

$$\bar{Y}... = \frac{1}{NMP}\sum_{1}^{N}\sum_{1}^{M}\sum_{1}^{P} Yijk$$

To estimate the population mean $\bar{Y}...$, a simple random sample of n first-stage units is selected. From each of the selected first-stage units, a simple random sample of m second-stage units is selected while from each of the selected second-stage units, a simple random sample of p third-stage units is drawn. Let $Xijk$ be the value of the auxiliary variable corresponding to $Yijk$, $Vijk = \dfrac{Yijk}{Xijk}$ and the quantities $\bar{X}ij.$, $\bar{X}i..$, $\bar{X}...$, $\bar{V}ij.$, $\bar{V}i..$ and $\bar{V}...$ be defined similarly.

Further, let

$$Sijrx = \frac{1}{p-1}\sum_{1}^{p} (Vijk - \bar{V}ij(p))\,(Xijk - \bar{X}ij(p))$$

$$Sirx = \frac{1}{m-1}\sum_{1}^{m} (\bar{V}ij(p) - \bar{V}i(mp)\,(\bar{X}ij(p) - \bar{X}i(mp))$$

$$Sbrx = \frac{1}{n-1}\sum_{1}^{n} (\bar{V}i(mp) - \bar{V}nmp)\,(\bar{X}i(mp) - \bar{X}nmp)$$

$$\bar{S}wrx = \frac{1}{n}\sum_{1}^{n} Sirx$$

$$\bar{\bar{S}}wrx = \frac{1}{nm}\sum_{1}^{n}\sum_{1}^{m} Sijrx$$

where $\bar{V}ij(p)$, $\bar{V}i(mp)$, $\bar{V}nmp$, $\bar{X}ij(p)$, $\bar{X}i(mp)$ and $\bar{X}nmp$ are the corresponding sample means.

Then an unbiased ratio-type estimate of $\bar{Y}...$ is given by

$$\text{Est } \bar{Y}... = \bar{V}nmp\ \bar{X}... + \frac{N-1}{N}\,Sbrx + \left[\frac{m-1}{m} + \frac{M-m}{Mm}\cdot\frac{1}{N}\right]\bar{S}wrx + \left[\frac{p-1}{p} + \frac{P-p}{Pp}\cdot\frac{1}{NM}\right]\bar{\bar{S}}wrx$$

If all the third-stage units in a second-stage unit have the same value, that is, $Yijn = \bar{Y}ij.$, we obtain the unbiased ratio-type estimate for two-stage designs, namely:

$$\text{Est } \bar{Y}.. = \bar{V}nm\ \bar{X}.. + \frac{N-1}{N}\,Sbrx + \left[\frac{m-1}{m} + \frac{M-m}{Mm}\cdot\frac{1}{N}\right]\bar{S}wrx$$

If, in addition, all the second-stage units in a first-stage unit have the same value, say, $\bar{Y}ij. = \bar{Y}i..$, we obtain the unbiased ratio-type estimate given by Hartley and Ross[1], namely,

$$\text{Est } \bar{Y}. = \bar{V}n\ \bar{X}. + \frac{N-1}{N}\,Sbrx$$

The general form of an unbiased ratio-type estimator for a t-stage design is now obvious and can be written down easily by induction.

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[*Note*.—It is understood that results similar to those recorded above have already been obtained by Dr. Alan Ross, and are to be published in a more comprehensive paper in due course.—EDITOR.]

[1] Hartley, M. U., and Ross, A., *Nature*, 174, 270 (1954).

MISCELLANEOUS

'Absolute' Age: a Meaningless Term

I WISH to appeal to my fellow geologists and workers in the rapidly growing subject of geochronology to discontinue a habit or fashion which is both unnecessary and misleading. I refer to the habit of calling radiometric ages 'absolute' ages. An age does not become 'absolute' by virtue of being expressed in units of time such as a year. If 'absolute' means anything at all, it implies complete independence of all events and relationships. But a year is a relationship and, moreover, our conviction that one year equals another is merely a convenient pragmatic hypothesis. As St. Augustine recognized long ago, there can be no real standard of time as there is of length or space, since the 'time' taken by an event passes away and cannot be brought back to be measured, as a metre rod can be measured by placing it alongside a standard metre rod. Yet we do not go out of our way to say that the 'absolute' length of the rod is one metre. Nor if you ask a man of forty-five how old he is, does he reply "My absolute age is forty-five years".

The term is not only redundant and both philosophically and scientifically without meaning: it is also misleading in its psychological suggestion of a higher degree of accuracy than can be justified. A good example has recently appeared in *Nature*[1], where the title of a particularly welcome contribution begins "Confirmation of the Absolute Age . . .". Now how, if an age is 'absolute', can it require confirmation?

If any of the culprits claim that they are in good company with Newton and Kelvin, who both wrote of 'absolute time', they have only to remember Einstein and the coming of relativity. When it is desirable, as it sometimes is, to distinguish between geological age and the age expressed in years, the latter (except when measured by counting varves) can appropriately be called the 'radiometric age', a term that has already appeared occasionally in the literature of geochronology.

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[1] *Nature*, 196, 665 (1962).