the following assumptions are more capable of justification:

Assumption 1: The derived probability distribution, in this instance Gaussian, must have its parameters chosen to give the highest probability to the observed data F.

Assumption 2: The derived distribution must have its parameters chosen to best approximate, in the sense of mean square error, the observed data F.

For different values of s the quantity p of equation (2) is plotted in Fig. 1 for the data of Nilakantan and Achar. Assumption 1 is of course best satisfied when $s = \sigma = 1.83$. The mean square difference between Y of equation (1) and F is also shown as a function of s in Fig. 1. It follows that assumption 2 is best satisfied when $s = 1.73$.

It is clear from Fig. 1 that the method of Nilakantan and Achar, which leads to $\sigma_9 = 1.29$, produces a probability distribution in very poor agreement with either of assumptions 1 and 2. This result, together with the fact that Nilakantan and Achar obtained experimental agreement with σ_3 rather than σ_9, leads me to conclude that their method should be used with considerable caution and only when there are sufficient samples available to permit a check on the validity of the assumed limiting process.

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[1] Nilakantan, P., and Achar, B. N., *Nature*, **193**, 1005 (1962).

Since when two samples from a population are combined we get a larger and more representative sample, the iterative process could, in effect, be considered as a means of obtaining a larger sample starting from a small one. As the distribution curve F_2 is derived from the mean values of the frequencies in F and F_1, that is, $(F + F_1)/2$, the result of the combination of two sets of data, namely, the actual curve obtained from n experiments and its Gaussian counterpart, may be regarded as being representative of $2n$ experimental observations (Fig. 1). On the same basis, we may regard F_3 (derived from F_2 and F_1) as representing $3n$ experimental observations and F_4, $5n$ experimental observations. Thus the sample size may be considered as increasing in the following sequence, namely, n, n, $2n$, $3n$, $5n$, ... $89n$, ... corresponding to F, F_1, F_2, F_3, F_4, ..., F_{10} and so forth. The $(n+1)^{\text{th}}$ term in this sequence is given by

$$n\left[\frac{1}{2}\left\{1 + \frac{1}{\sqrt{5}}\right\}\left\{\frac{1+\sqrt{5}}{2}\right\}^n + \frac{1}{2}\left\{1 - \frac{1}{\sqrt{5}}\right\}\left\{\frac{1-\sqrt{5}}{2}\right\}^n\right]$$

We have since found that a more rational procedure would be to use $\dfrac{nF + nF_1}{2n}$ instead of $\dfrac{F + F_1}{2}$ to get F_2,

$\dfrac{nF_1 + 2nF_2}{3n}$ instead of $\dfrac{F_1 + F_2}{2}$ to get F_3, and so forth.

The results obtained by this modification are, however, not significantly different from those obtained previously. But the latter procedure may be considered more logical as indicative of the increase in equivalent sample size.

The iterative process is seen to converge slowly at first, but more rapidly later as may be expected, since the 'equivalent' or effective sample size also increases.

Prof. Heaps has assumed that the parameters of the Gaussian distribution curve must be so chosen that they should either give the highest sample probability
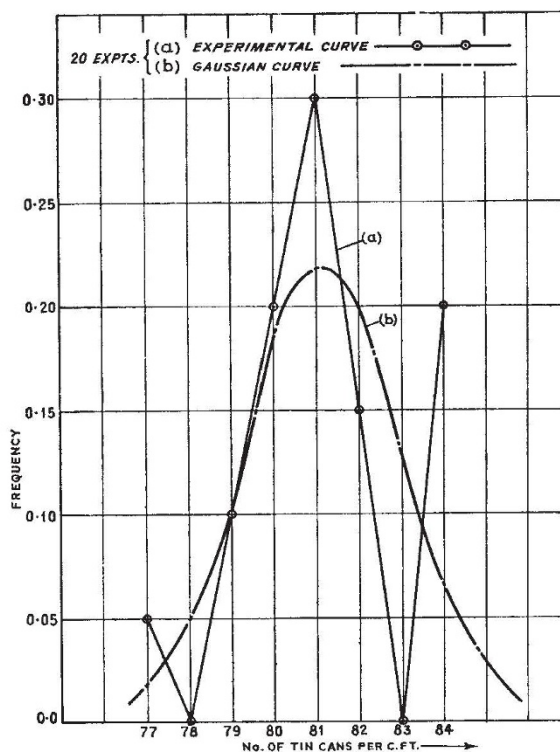

Fig. 1. Frequency distribution curves

to the observed data or form the best approximation to the observed data in the sense of mean square difference. He finds that in the case of the experiments carried out by us, $s = 1.83$ would give the highest sample probability for the observed data for 20 experiments, whereas $s = 1.73$ would give the least mean square difference. As σ_9 obtained from the iterative method has a value 1.29, which is different from either of these, he concludes that the method should be used with caution.

As the Gaussian distribution curve, F_1, has been fitted to the observed data, F, by obtaining the standard deviation $\sigma = 1.83$ from the observed data, the first assumption made by Heaps is self-evident, as this value maximizes the sample probability function 'p' which is a characteristic of the size of the sample considered, that is, 20 in the present case. However, in the iterative method the sample size should be regarded as increasing with successive iterations and different 'p' functions would have to be used. Hence $\sigma_9 = 1.29$, which would maximize the 'p' function corresponding to an 'effective' sample size of 1,780, is not inconsistent with $\sigma = 1.83$ maximizing 'p' for a sample size of 20. The difficulty experienced by Heaps in fitting the results of the iterative method with his assumptions is thus understandable since the variation of the effective sample size was not taken into account by him.

We agree that a considerable amount of experimental investigation will be necessary to establish the general applicability of the method and its possible limitations. We are continuing our investigations on these aspects.

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