t is finite. The equation is only an idealized approximation: it would be better enunciated for the large but finite number N_0 of atoms in a large but finite initial volume V_0: for all that, the equation is not objectionable. But the substitution of an infinity for t is objectionable. The infinity to which t tends is not $\aleph_0$. In Dr. Schlegel's formula :

$$N \geqslant e^{\aleph_0} \quad (2)$$

the right-hand side is meaningless: and the argument does not yield:

$$N \geqslant 2^{\aleph_0}$$

as Dr. Schlegel claims. It yields only that $N \geqslant K$ for any finite integer K, that is:

$$N \geqslant \aleph_0 \quad (A)$$

The hypotheses involved here are that the universe is expanding, and that it looks the same everywhere at all times.

To obtain the opposite inequality:

$$N \leqslant \aleph_0 \quad (B)$$

one is much less committed. Irrelevant are all questions of isometric embedding: relevant only is the question whether the universe can be exhausted by a countable set of small neighbourhoods: this is in essence the question whether the universe is 'separable'. There exist (in the mathematical sense) topological manifolds which are not separable, but that sort of thing is excluded by the assumption of a global metric: it is a familiar fact that any Riemannian manifold is separable. Even if we abandon the hypothesis of a Riemannian metric in some quantized theory, it can scarcely be doubted that (B) would still follow from the steady-state hypothesis, in any event if this included some hypothesis of isotropy.

On the usual interpretation of the red shift we can have no future physical connexion with atoms beyond a certain finite distance. This leads to a difficulty about the derivation of (B) which may trouble some. Do such atoms 'exist'? How can they be 'counted'? Such questions invite confusion between conceptual existence (on one of the steady-state-expanding-universe concepts) and physical-existence-for-us: and doubt may easily result. But the universe consists of those atoms, and parts of space, which are connected with us not necessarily by one such physical connexion but by a finite chain of such physical connexions as are envisaged (reception and transmission of signals). This gives a closed system: to conceive the 'existence' of any further atoms is to indulge in idle fantasy: this closed system is the universe: and it is separable.

H. D. URSELL

Department of Mathematics,
University of Leeds.

[1] *Nature*, 193, 665 (1962).

THE application of transfinite numbers to physical theory is admittedly novel, and should be open for critical discussion; none the less, I see no force in the arguments which Mr. Ursell presents.

In my communication I implicitly made use of the accepted relation $2^{\aleph_0} = n^{\aleph_0} = C$, where n is any integer greater than 1 and C is the power of the continuum, and on the basis of this relation I set $e^{\aleph_0} = C$. It is true that $e^{\aleph_0}$ has not been defined in transfinite number theory; we may, however, readily come to the non-denumerability of atom-spaces, in steady-state theory, without making use of this quantity. The growth equation $N = N_0 e^{kt}$ requires $N/N_0 = 2$ for a time-interval $t_2 = (1/k)\log 2$. It follows, then, that any chosen finite set of atom-spaces will be doubled in time t_2. Indeed, over-looking fluctuations in the presumed creation process, we can say that in time t_2 each atom-space becomes two atom-spaces. Hence, each atom-space in a given set becomes two atom-spaces in an interval t_2. Since, by hypothesis, the universe has existed for a time $\aleph_0$, and $\aleph_0 \cdot t_2 = \aleph_0$ (we may choose time units so as to make t_2 an integer), we have a set of $\aleph_0$ factors of 2. Hence, a set of atom-spaces with cardinal number $2^{\aleph_0} = C$ must have been formed in the universe. We could equally well have worked with a t_n, defined for an n-fold increase of any finite group of atoms, with the same result: there is a set of atom-spaces with cardinal number $n^{\aleph_0} = C$.

It is a basic tenet of steady-state theory that the universe has always existed with its present large-scale features. I take this to mean that the universe has existed for an achieved infinity of years: a set of years with the cardinal number $\aleph_0$. The alternative assumption is that the life-time of the universe is t years, where t 'tends to infinity', that is, is indefinitely large, but is still a finite number. This assumption would not give the universe the steady-state property of having existed for ever.

Mr. Ursell proposes that cosmological considerations must be limited to those parts of the universe with which physical interaction, as through signal chains, is possible. Such interaction is a factor of doubtful relevance in the consideration of the consistency properties of cosmological models (as opposed, of course, to matters of empirical confirmation). In any event, he presents no argument that would allow a set of C atom-spaces to be fitted into a universe which everywhere has the spatial properties (in accordance with the cosmological principle used in steady-state theory) that we find in our observed universe.

RICHARD SCHLEGEL

Department of Physics,
Michigan State University,
East Lansing, Michigan.

THE force of my arguments must, it seems, be a matter of opinion. Dr. Schlegel "may readily come" to various conclusions, but mostly without me.

Dr. Schlegel's central fallacy, on my evaluation, lies in the easy substitution of an infinity for a finite number in a formula. This is how he leaves the slippery slope of ambiguities for the abyss: this is where I do not follow. In the foregoing communication the error is implicit in his phrase, "set of $\aleph_0$ factors", explicit in the next sentence. I concede a factor of 2^n between the year n B.C. and now: I do not concede his factor of $2^{\aleph_0}$ between, presumably, the cold bleak world of $\aleph_0$ B.C. and now.

Dr. Schlegel's objection to my final paragraph is disheartening. I am there defending, against some quite possible criticism, the statement (B) above which he himself made (ref. 1, p. 666, line 12). Even when I agree with him, he disagrees with me.

H. D. URSELL