While in some other species, as in the rat, both types of α cells are equally well stained with phloxine or ponceau fuchsin, it is worthy of note that the α_1 cells in the dog pancreas have lower affinity to these stains. This might well explain why the uncinat process has been considered erroneously to be devoid of α cells in spite of the relatively very high contribution of α_1 cells demonstrated by us in the islets of this particular region. The finding that the uncinat process of the dog pancreas was characterized not only by extremely low or absent extractable hyperglycaemic activity but also by a different cellular composition of the islets due to absence of α_2 cells suggests that this latter type of α cell is the source of glucagon. The recent demonstration¹⁰ of pronounced regressive nuclear changes in the α_2 cells after repeated injections of glucagon to rats supports such a hypothesis.

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STATISTICS

Spherical Errors

I HAVE read with interest the recent article by N. Singh¹ concerning spherical errors. Work of a similar nature has been done in this laboratory.

It is assumed here that the components of the total errors in the position of an object along three cartesian co-ordinates (x, y, z) are normally distributed with variances $\sigma_x^2, \sigma_y^2, \sigma_z^2$, and mean at the origin. That is, the probability density of the position of the object in question along the x co-ordinate is:

$$\frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{x^2}{2\sigma_x^2}\right]$$

and similarly for the other co-ordinates. The probability $P(R)$ that the object is within a sphere of radius R about the origin is then given by:

$$P(R) = \frac{2\pi^{-3/2}}{\sigma_x \sigma_y \sigma_z} \iiint_R \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right] dx dy dz$$

where the integration is carried out over the volume of the sphere in question. Unfortunately, this expression for $P(R)$ cannot, in general, be integrated in closed form, nor is it a tabulated function.

In order to avoid the difficulty of a triple numerical integration, it is usual to assume that $\sigma_x = \sigma_y = \sigma_z = \sigma$, whence the expression for $P(R)$ becomes (after conversion to spherical co-ordinates):

$$P(R) \approx \frac{\pi^{-1/2}}{\sigma^3} \int_0^R \exp\left[-\frac{r^2}{2\sigma^2}\right] r^2 dr$$

If the substitution $r^2/2\sigma^2 = u$ is made, this can be expressed as

$$P(R) \approx \frac{2}{\sqrt{\pi}} \int_0^{R^2/2\sigma^2} e^{-u} u^{1/2} du$$

which is the incomplete Γ function², and is tabulated.

In making the aforementioned assumption, personnel normally choose a value for σ^2 which is equal to the sum of the variances of all the errors involved, regardless of their direction. This naturally results in a value of $P(R)$ which is smaller than the actual value. The transition from ellipsoidal to spherical symmetry suggests that a better alternative than the above might be to assume:

$$\sigma^2 = \frac{1}{3} (\sigma_x^2 + \sigma_y^2 + \sigma_z^2)$$

As the values of σ_x, σ_y , and σ_z diverge, the spherical approximation to the real case becomes less valid. It is therefore worth while to establish some limits on the actual value of $P(R)$ to increase the confidence in any approximate answer obtained. On examination of the expression for $P(R)$ it is obvious that:

$$\frac{2}{\sqrt{\pi}} \int_0^{R^2/2\sigma_M^2} e^{-u} u^{1/2} du \leq P(R) \leq \frac{2}{\sqrt{\pi}} \int_0^{R^2/2\sigma_m^2} e^{-u} u^{1/2} du$$

where σ_M is the maximum of ($\sigma_x, \sigma_y, \sigma_z$) and σ_m is the minimum of ($\sigma_x, \sigma_y, \sigma_z$). Thus, a set of limits on the actual value of $P(R)$ is established.

Another set of limits may be established by integrating the expression for $P(R)$ over the volume of a cube rather than a sphere, that is:

$$\begin{aligned} \frac{2\pi^{-3/2}}{\sigma_x \sigma_y \sigma_z} \int_{-R/\sqrt{3}}^{R/\sqrt{3}} \int_{-R/\sqrt{3}}^{R/\sqrt{3}} \int_{-R/\sqrt{3}}^{R/\sqrt{3}} \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right] dx dy dz &\leq P(R) \\ &\leq \frac{2\pi^{-3/2}}{\sigma_x \sigma_y \sigma_z} \int_{-R}^R \int_{-R}^R \int_{-R}^R \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right] dx dy dz \end{aligned}$$

The two cubical regions of integration are taken as the cubes that circumscribe and inscribe the actual region of integration. The limits are then given by the product of three tabulated functions (error functions). This latter set of limits is the more useful when the values of the variances are widely divergent. Taking the larger of the two possible lower limits and the smaller of the two larger limits should establish a fairly good bracket on the value of $P(R)$. A graph of the limits of $P(R)$ versus R then establishes limits for the radius of the 50 per cent probability sphere.

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² Pearson, K., *Tables of the Incomplete Γ Function* (Camb. Univ. Press).