typical journal article, they are, by and large, slighter. As a consequence, they are for the most part easy to read. But it would be a hopeless task to attempt to review all of them, or even to try to pick out objectively the most important. All I can do is to mention some that caught my fancy. These include the two offerings from England, by M. G. Kendall and M. S. Bartlett, the latter discussing his concept of quasi-stationarity which promises to be, in one form or another, a key idea in the future development of stochastic process theory. Bradley and Prendergast extend modern ranking theory which already owes much to the former author. Block and Marshak have an intriguing paper on a cognate topic. Roy and Bhapkar expound and develop the Pearson–Roy logical analysis of multiway contingency tables. But perhaps the tributes which do Prof. Hotelling most honour are the papers by Bowker and Wilks developing classical multivariate analysis, and that of Stein, who looks at it from the most recent Californian point of view.

D. E. BARTON

ANALYSIS ON RIEMANN SURFACES

Riemann Surfaces

By Prof. Lars V. Ahlfors and Prof. Leo Sario. (Princeton Mathematical Series, Vol. 26.) Pp. x+382. (Princeton, N.J.: Princeton University Press; London: Oxford University Press, 1960.) 80*s*. net.

THE modern era of Riemann surfaces was ushered in by H. Weyl's book *Die Idee der Riemannschen Fläche*. Here we find for the first time the systematic development of the idea of an abstract Riemann surface as an analytic manifold of one complex dimension, essentially a space with local parameters related in such a way that two distinct parameters are regular functions of each other in their common region of definition. Since the Second World War there has been a tremendous development in the field and books have appeared by Nevanlinna, Pfluger, Schiffer and Spencer, and Springer, for example.

Ahlfors and Sario's *Riemann Surfaces* goes further than its predecessors. The first chapter contains all the necessary topology which plays a fundamental part in the theory. The next chapter introduces Riemann surfaces and some of the basic methods for treating them, subharmonic functions, and the problem and integral of Dirichlet, Rado's theorem about the separability of Riemann surfaces which follows in a particularly simple and elegant way, and the fact that a smooth Riemannian metric on a surface induces an essentially unique conformal structure, a result which is frequently stated but seldom proved. The third chapter treats functions with singularities on a surface by Sario's elegant method of normal operators and deduces the principal mapping theorems for planar surfaces, that is, those topologically equivalent to plane domains. Such surfaces can be mapped on a plane domain bounded by various kinds of slits. In the simply connected case, we obtain the closed or open plane or the unit disk.

The second half of the book consisting of the last two chapters is more for experts. The fourth chapter introduces a complete exposé of the modern theory of degenerate surfaces, that is, those not admitting functions of various kinds and their inter-relations. This subject was opened by Ahlfors in the late 'forties and the solutions of the problems raised by him have now been almost completely obtained by the authors of the present book and others. The last chapter treats differentials on arbitrary Riemann surfaces and finishes with a welcome section on the classical theory of integrals on closed surfaces and the Riemann–Roch theorem.

This is an authoritative and long-awaited treatise by two of the greatest experts in the field. The elegant formulation of many of the ideas and proofs have made it possible to include more material than there is in any other book on the subject. The presentation is fairly clear but the subject is difficult, and particularly in the latter chapters the approach is very abstract. If one wishes to have complete generality and at the same time prove the classical theorems, it is most economical in time and space to deduce the latter from the former; but it is not necessarily the easiest way for the reader. Although this book is completely self-contained, I should recommend it rather to those readers who already know something of the subject—having looked, for example, at G. Springer's *Introduction to Riemann Surfaces*—than to the complete tyro. As a reference book, however, and a complete survey of all techniques right up to the frontiers of the subject, it is unequalled, and every serious mathematical library should contain it.

W. K. HAYMAN

MODERN GEOMETRY

Lectures on Modern Geometry

By Beniamino Segre. (Consiglio Nazionale delle Ricerche—Monografie Matematiche, 7.) Pp. xv+479. (Roma: Edizioni Cremonese, 1961.) 7,000 lire.

MODERN geometry is a term which has seen service in many campaigns and under many commanders. However, in the present instance, the use of the title is not only justified at the moment but also stands a good chance of remaining so for some time to come; for this book, which is a greatly enlarged version of an Italian original of 1948, completes in various respects a generalization which has been under way for centuries.

Algebraic geometry as we know it to-day stems from two main sources: the invention of co-ordinates, and the analysis of axiomatic structure. The latter process has given a whole series of shocks to intuition; these, once absorbed, have led to the creation of new geometries which find their natural expression in terms of suitable co-ordinate systems. In a definitive account this process may well be reversed; in Prof. Segre's work we have examples of both procedures.

Part I of the book is a lucid exposition of the algebraic concepts and results which are required later. A useful innovation here is the term corpus to denote a general field, the latter name being reserved for commutative corpora. Part 2 is devoted to the foundations of projective geometry over an arbitrary corpus. The first chapter deals with linear spaces, that is, geometry based on homogeneous point co-ordinates. In the second chapter, which is concerned with graphic spaces, the approach is axiomatic: a graphic space is any set of abstract elements ('points') which satisfy the incidence relations of projective geometry. On this basis one proceeds to erect a co-ordinate system.

At this point Desargues' theorem on triangles in perspective comes into play. It can be shown that the theorem holds in every linear space, but that there are graphic planes in which it is false. In contrast